

DECISION MAKING BY ESTIMATING MULTIPLE LOOP SYSTEM
REACTION IN AN EXCHANGE SYSTEM

A THESIS

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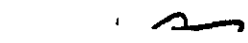
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SUMMARY

In the real world, there exist shifts in policy on the part of the nation, the company, or the organization. In this study by using a decision-making function, each participant in the exchange system can estimate multiple-loop system reaction from the various possible actions available to him.

Each participant determines his most efficient action based on the estimated reaction from each possible action under the progressive situation. Then each participant--nation, company, or organization--gradually shifts his policy to get the most efficient reaction for his purpose.

As an example of the type of problem to which this modeling method can be applied, a three-person reciprocal language learning model was discussed. Simulations were run on a large-scale digital computer, using Dynamo, to produce the time response of variables in the model.

This study suggests that this modeling method helps to explain and characterize the decision-making process in an exchange system.

CHAPTER I

INTRODUCTION

Background

A considerable amount of human activity is conducted within the context of an exchange system: a number of participants act toward one another for mutual benefit or harm. The actions that one participant receives motivate him to react, or act toward the other participants. Each participant normally attempts to act in such a way that the reactions he receives will provide him the greatest possible satisfaction.

This general setting of participants and actions must include a consideration of the participant's needs, which change with time. The needs of each participant may be influenced by the actions of the others, giving a closed loop feedback structure. In other instances the needs may be determined largely by exogenous factors, leading to an open loop structure for purposes of analysis.

Another, perhaps more serious, complication is the fact that each participant must predict how the others will react to the various possible actions available to him. Will an action by one participant induce the same reaction in another at different points in time, given that the needs

are the same? What is the possibility of negotiating mutually beneficial actions?

Related to this last issue is the complexity of inter-relationships possible. For example, in a system involving four participants, A, B, C, and D, participant A may act towards B, who may react towards A or towards C. Participant C in turn may react towards A or D, with the latter again reacting. This process can continue forever, but the human mind must stop somewhere in considering all the chain reactions.

Little work has been done to model decision making in such dynamic, multi-product exchange systems. Mathematical game theory is difficult to apply because it deals with essentially static situations that can be described by a discrete number of alternatives and outcomes. Economic multi-product market equilibrium models make the implicit assumption of an impersonal market where any product (or action) may be bought or sold with money, or a monetary standard of numeraire.

Yet, virtually all cooperative and competitive behavior among people occurs within the context of such dynamic exchange systems, from the small-scale example of two students helping each other with assignments to the large-scale international conflicts involving oil and military strength.

Purpose of the Thesis

It is the purpose of this research to develop and study some analytic models of decision making within the context of dynamic multi-product exchange systems. Throughout, it will be assumed that each participant acts to maximize the satisfaction he derives from the other participants' reactions.

In particular, the following will be examined:

1. Consideration of the other participants' needs in the decision making process.
2. Methods for predicting reactions from the other participants in the system.
3. A framework for attaching different levels of importance to more and more complex reactions within the system.

It is anticipated that general results concerning the behavior of the system will be obtained for each of the above items.

Method of Approach

The methodology to be used will be fixed-increment simulation of a general exchange system model. This general model, described in Chapter III, was selected as a plausible model suitable for analyzing the effects of decision making procedures. It attributes to each participant four basic functions:

1. Situation Function to measure the value of the actions received from the other participants.

2. Motivation Function to determine the total amount of effort to produce action.

3. Decision Function to estimate the reactions expected for each possible action.

4. Action Function to determine the amount of each action to take.

Concentrating on the Decision Function, three ways of estimating reactions of participants will be examined:

1. Using observed values of how each participant splits his effort among his possible actions.

2. Using these same observed values and forecasting linearly based on recent trends.

3. Considering the mutual benefit each pair of participants may derive from two-way exchanges.

The effect of considering different levels of complexity for the reactions will be studied by varying the attached importance levels.

The simulation will be performed on the Georgia Tech UNIVAC 1108 using DYNAMO simulation language. Originally developed for Feedback Dynamics analysis, the DYNAMO language is appropriate for this research because it is a convenient fixed-increment simulation language, that easily handles first order difference equations.

CHAPTER II

REVIEW OF ALTERNATIVE APPROACHES

This chapter reviews briefly some other analytic approaches that might be used to model dynamic, multi-product exchange systems. These are market equilibrium models, game theory, and feedback dynamics. The shortcomings of these techniques for achieving the purpose of this research will become evident in the discussion.

Market Equilibrium Models

Economic multi-product, multi-firm market equilibrium models can be used to describe the situation of m firms producing and consuming n products. Each firm i has a demand curve h^{ji} for product j , where x is a price vector for the m products. If the firm is a producer of product j , then $h^{ji}(x)$ is negative; if a consumer, then $h^{ji}(x)$ is positive. In general, the partial derivative of $h^{ji}(x)$ with respect to x_j , the price of product j , is negative:

$$\frac{\partial h^{ji}(x)}{\partial x_j} < 0 \quad (\text{II-1})$$

At higher prices consumers demand less and producers supply more [7].

Under certain conditions regarding the cross elasticities of the demand curves the system can be shown to converge to an equilibrium where the price vector x remains constant. Lag adjustment models can be used to describe transient behavior while converging.

These market models standardize on one product, say product 1, as the monetary standard or numeraire. Setting $x_1 = 1$ allows all other x_j to be determined from partial derivatives.

$$\frac{\partial h^{ji}(x)}{\partial x_j} / \frac{\partial h^{li}(x)}{\partial x_1} = x_j / x_1 \quad i = 1, \dots, m \quad (\text{II-2})$$

in the equilibrium.

Herein lies the difficulty of applying market models to general exchange systems. The above partial derivatives (II-2) may be non-existent or ill-behaved. Furthermore, in a general exchange system the ratios $\partial h^{ji}(x) / \partial x_j$ depend upon who does the transacting.

In other words, military action from one Middle Eastern country against another may be of no consequence to a Southeast Asian country. There is no market where military, political, and oil actions may be traded freely. The trading depends upon who does the acting.

It is true that negotiation may occur among three or more participants in such situations. This results in

discounting of the value of certain actions by some participants. No formal way has yet been devised to include this type of behavior in a market model.

Game Theory

Mathematical game theory is useful for describing conflict situations involving n participants, each of whom has available to him a limited number of actions n_k . The payoffs to the participants are described by a matrix of size $n \times n_1 \times n_2 \times \dots \times n_n$. The mathematics behind game theory are related to mathematical programming.

This type of analysis is designed for static, repetitive situations where all participants act simultaneously. Virtually all of the mathematics is based on the assumption that each participant tries to optimize, under restrictions, his short-term payoff. There is no consideration of time-delayed reactions from the other participants. Game theory thus does not lend itself to describing dynamic systems.

Feedback Dynamics

Feedback dynamics is a methodology for modeling and analyzing the behavior of a broad range of dynamic systems. The approach begins with observing the dynamic behavior of the problem variables, constructing relationships between the variables and building a model using non-linear, first-order difference equations. The components of the system are

represented as accumulations, goals, forces, and time rates of flow. The analysis focuses on the positive and negative feedback loops within the system.

A feedback dynamics analysis of decision making in a general exchange system, while appropriate, would be beyond the scope of this research.

Such a task would involve a considerable amount of empirical work before any model formulation could be done. The analysis of the feedback loops within the system and how they influence the system behavior would also be divergent from the purpose of this research, which is to study the decision making aspect only.

Accordingly, the general model developed in Chapter III is designed to be a plausible model suitable for this purpose. The model contains accumulations, goals, etc., along with other components specific to various aspects of the information gathering and decision making process.

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CHAPTER III

DESCRIPTION OF A MODEL OF PARTICIPANTS
IN A SYSTEM

A system is the aggregation of many kinds of participants, and the system is defined by determination of the relationship among these participants.

When we observe the relationship between one of the participants and another in the system, we see that the participant is taking some kind of and some amount of action upon the system and receiving some kind of and some amount of reaction from the system. (Refer to Figure 3-1.)

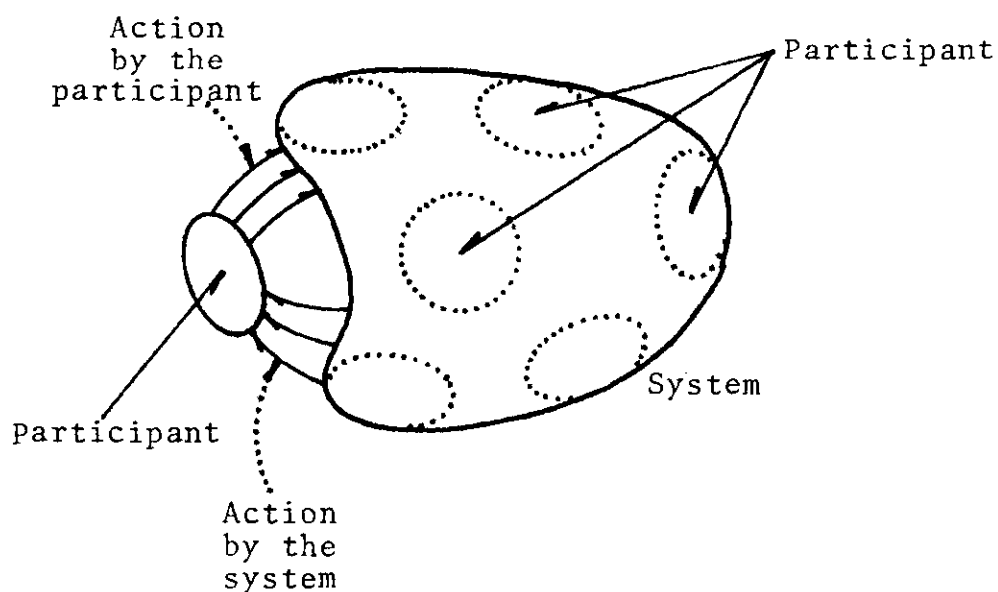


Figure 3-1. Relationship Between One of the Participants and the Other

The participant is determining the kind of and the amount of action, so that he can get the highest level of satisfaction.

This attitude is observed easily in the decision making involving diplomatic policy by the nations, in the decision making regarding a company's policy concerning the customer or competitor, and the decision making of every kind of organization. For example, in the case of decision making by a nation, the nation will decide the kind of and amount of action, political support, economic cooperation, and trading relations in order to maximize her own satisfaction.

The process in which the participant determines his actions is as follows (see Figure 3-2):

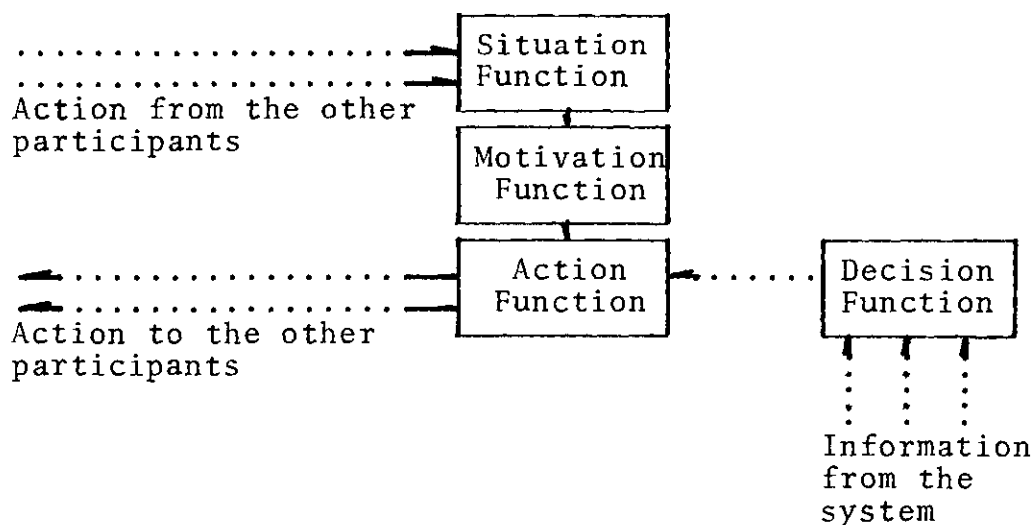


Figure 3-2. Mechanism of the Determination of the Action

A Situation Function embodying specific value judgment measures the value of the actions which the participant is receiving from the other participants in the system. The value of the actions which is calculated by the situation function is the basis for the motivation for the actions. In turn, the Motivation Function influences the actions one participant takes toward the others.

The Decision Function collects all kinds of information from the participant in the system and estimates the system reaction from the various possible actions available to him. Then the Action Function determines his most efficient actions, based on the estimated reaction, to get the most efficient reaction for improving his situation. Details of these functions will be discussed in this chapter.

Situation Function

The Situation Function determines the value of the actions which the participant is receiving from the other participants in a system.

If it is assumed that n participants exist in a system and each participant is receiving only one kind of action from each participant, then the situation S_{pa} of a participant P_a ($a = 1, 2, \dots, n, a \neq b$) is given by the following equation:

$$S_{pa} = f(A_{1a} X_{1a}, A_{2a} X_{2a}, \dots, A_{ba} X_{ba}, \dots, A_{na} X_{na}) \quad (\text{III-1})$$

$$b = 1, 2, \dots, n \quad (b \neq a)$$

In the above equation, X_{ba} is the action by a participant P_b upon a participant P_a , and A_{ba} is the Need Coefficient of the participant P_a for the action X_{ba} .

In this research, it is assumed that this function is an additive function. Therefore, the equation will be as follows:

$$S_{pa} = A_{1a} X_{1a} + A_{2a} X_{2a} + \dots + A_{ba} X_{ba} + \dots + A_{na} X_{na} \quad (\text{III-2})$$

$$b = 1, 2, \dots, n \quad (b \neq a)$$

This equation says that an action where the need coefficient has a large value contributes to an increase in the value of the situation much more than an action where the coefficient has a small value in the Situation Function. The participant has the motivation to increase the value of the Situation Function most efficiently.

Motivation Function

The Motivation Function determines the total amount of effort which the participant will expend to produce action in the system.

In Figure 3-1, we observed that the participant is taking some kind of and some amount of action upon the system and receiving some kind of and some amount of reaction from the system (see Figure 3-1). When the system increases the amount of action upon the participant, the participant will

increase his effort; this behavior on the part of the participant is the result expected by the system, due to the promise or commitment of the participant to the system.

Homans [5] summarized this mechanism:

Social behavior is an exchange of goods, material goods but also nonmaterial ones, such as the symbols of approval or prestige. Persons that give much to others try to get much from them, and persons that get much from others are under pressure to give much to them.

That is, the participant can receive the actions from the system under the promise to react to the system in proportion to the action which he is receiving from the system.

But the participant judges the value of the system's action by his own situation; the value of the Need Coefficient A_{ba} in the Situation Function can be negative if this action is disliked by the participant, and the system's action with a negative Need Coefficient makes the value of the Situation Function decrease. But when the Motivation Function is considered, the important factor is the degree of closeness of the relationship with the system no matter whether the value of the Situation Function is negative or positive. This degree of closeness is defined by the sum of the absolute value of the terms in the Situation Function. That is, the negative terms in the Situation Function also cause the Motivation Function to increase.

The Motivation Function is given as follows:

$$Mpa = |A1a X1a| + |A2a X2a| + \dots + |Aba Xba| + \dots + |Ana Xna|$$

(III-3)

On the other hand, even though the value of the Situation Function is zero, the motivation for the action still exists, that is, some kind of factor which is not caused by the result of the action by the system will be the motivation. Therefore, in general the Motivation Function should include a constant term as shown:

$$Mpa = \Sigma |Aba Xba| + C$$

(III-4)

Action Function

The Action Function determines the actual action which the participant has to take based on the result from the Decision Function. The value of the Motivation Function is distributed by the Distribution Coefficient Kab , which is given by the Decision Function. The Decision Function will be discussed in detail in a later part of this chapter.

The actual action is given by the following equation:

$$Xab = Mpa \times Kab \times Bab$$

(III-5)

In this equation, Bab is the Transformation Coefficient which includes the efficiency of the effort to the action and coefficient to adjust to the real dimension of

the action.

The following diagram is a description of the mechanism of the Situation Function, Motivation Function, and Action Function. (See Figures 3-3, 3-4.)

Decision Function

The Decision Function determines the Distribution Coefficient K_{ab} which is used by the Action Function.

By use of the Decision Function, the participant can estimate system reaction from the various possible actions available to him. He determines the Distribution Coefficient which distributes his effort so that he can receive the most favorable reaction from the system.

First, we shall discuss the case in which three participants exist in a system (see Figure 3-5).

In this case, it is assumed that each participant is receiving only one kind of action from each of the other participants.

As a result of the analysis made so far, this model can be described as follows (see Figure 3-6).

The Decision Function of each participant is to collect from all parts of the system all the information necessary to understand his own situation. Consideration of the delay or modification of information should be required under the real problem, and the Decision Function recreates the model based on these delayed and modified information

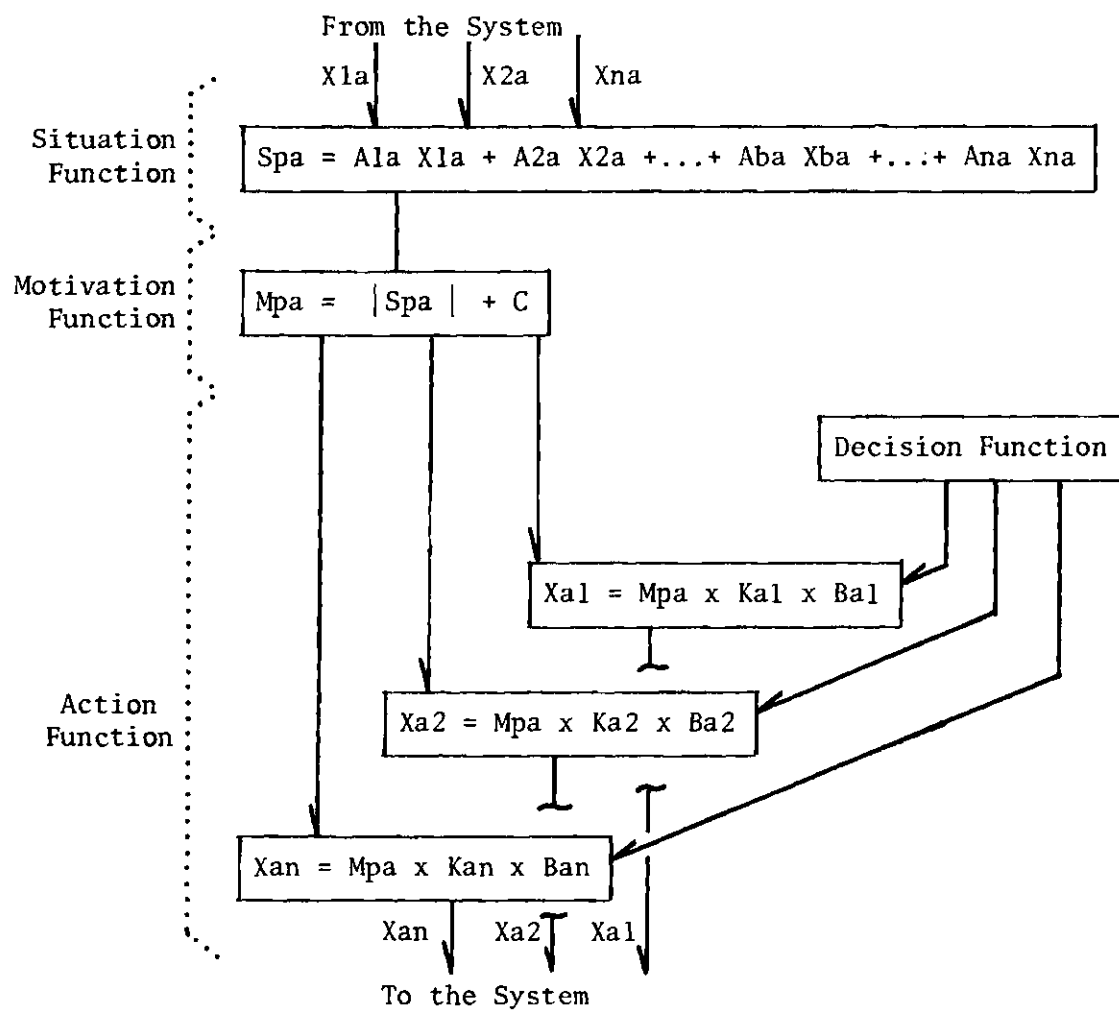


Figure 3-3. Schematic Describing the System

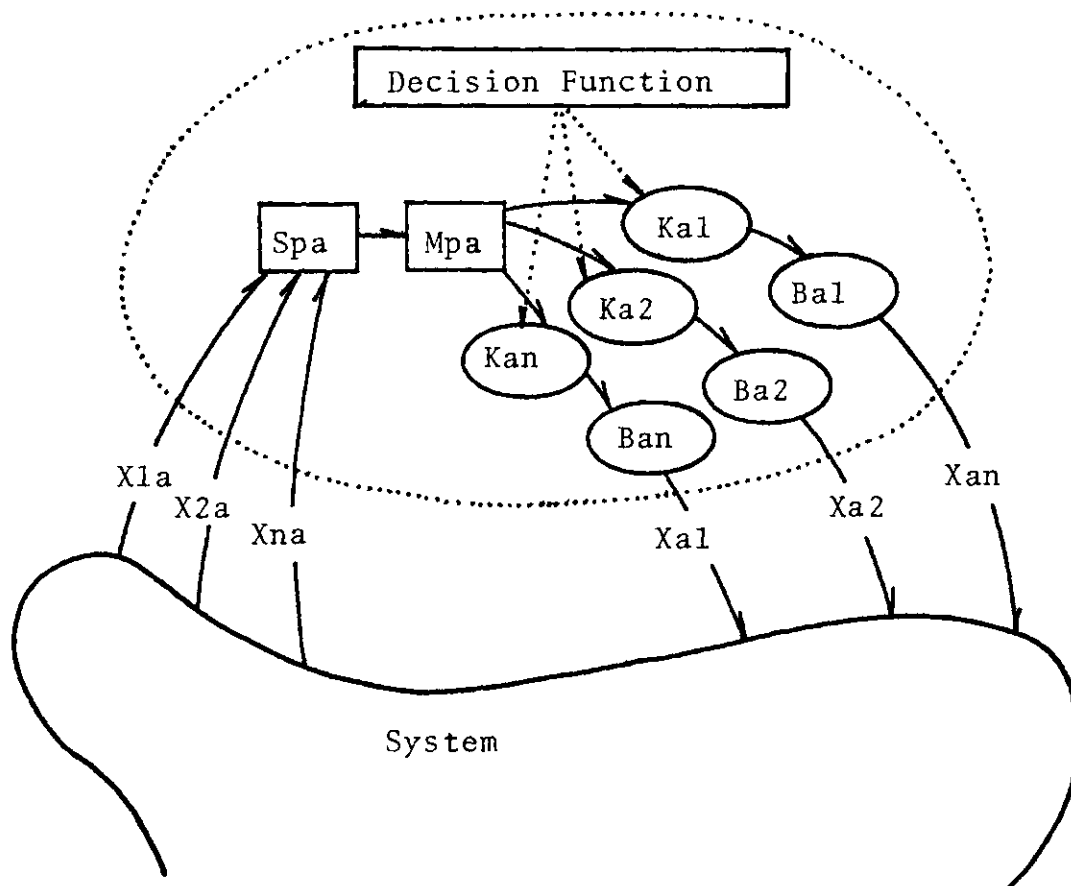


Figure 3-4. Simplified Schematic Describing the System

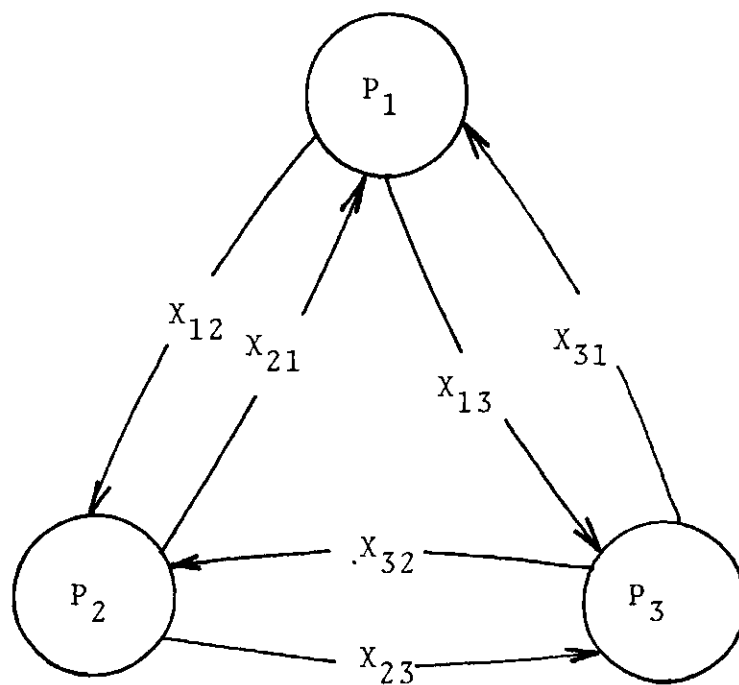


Figure 3-5. Relationship of Three Participants in a System

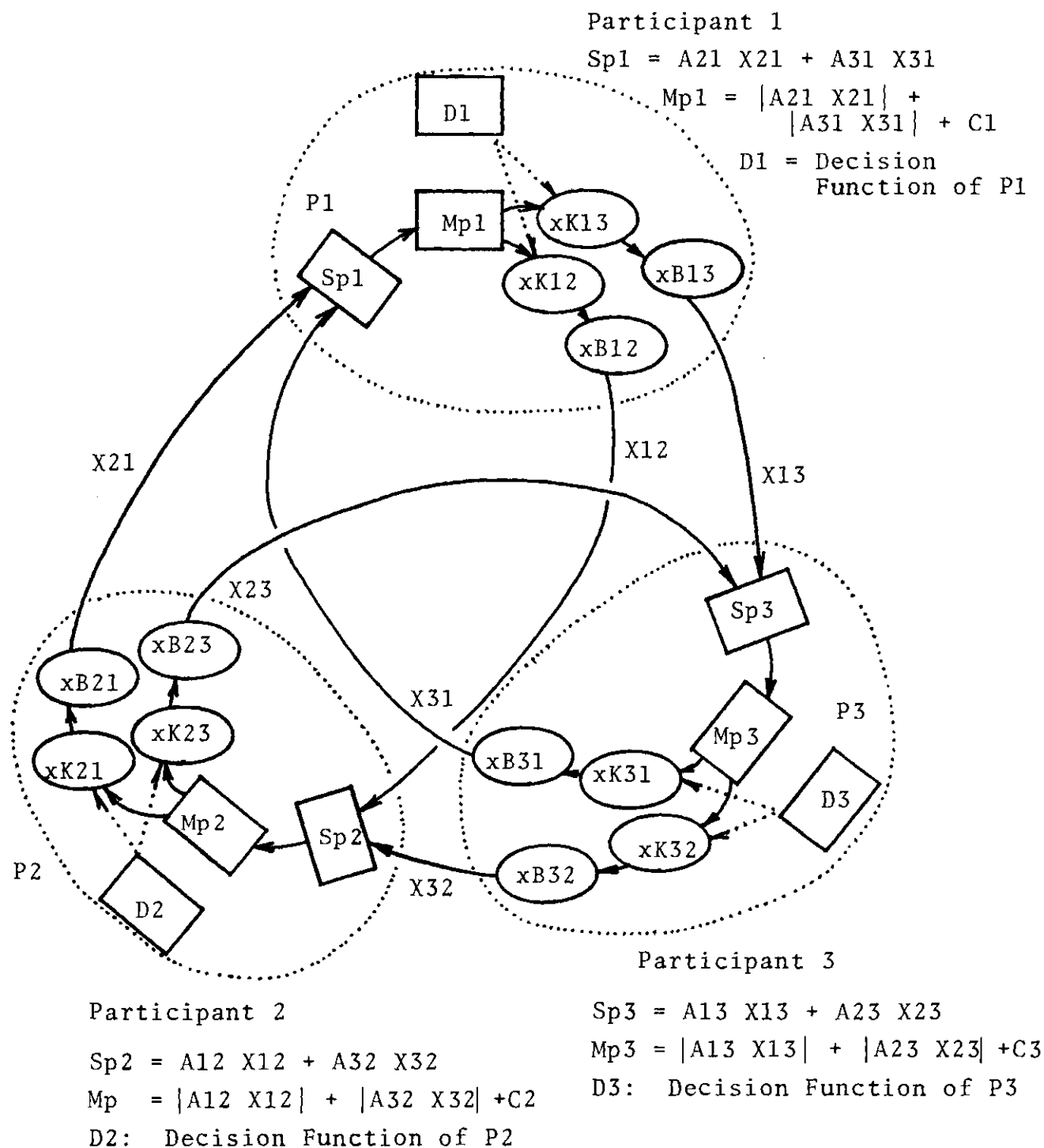


Figure 3-6. Model of Three Participants in a System

inputs. This recreated model in Decision Function is called here the Perceived Model.

Based on this Perceived Model in the Decision Function of each participant, he will estimate system reaction from the various possible actions.

The relationship of the actual model and the model in a case of three participants is described graphically as follows. (See Figure 3-7.)

The general attitude of the participant in a system at the point of decision making is that each participant must consider how the influence of his decision making will be realized in a system and how much of the value of the reaction in his Situation Function he will be able to receive finally.

In the case of participant P1 in Figure 3-6, he has two possible actions, X12 and X13, therefore, he must consider the influence of these two available actions, X12 and X13.

If it is assumed that participant P1 considers only two-way loops and three-way loops for each of his actions, action X12 has two loops, P1-P2-P1 and P1-P2-P3-P1; in the same way, action X13 has two loops, P1-P3-P1 and P1-P3-P2-P1. Through these loops, participant P1 can compare the influence of his actions X12 and X13.

The amount of change which is produced by one unit of effort for X12 in the value of the Situation Function of participant P2 is given as:

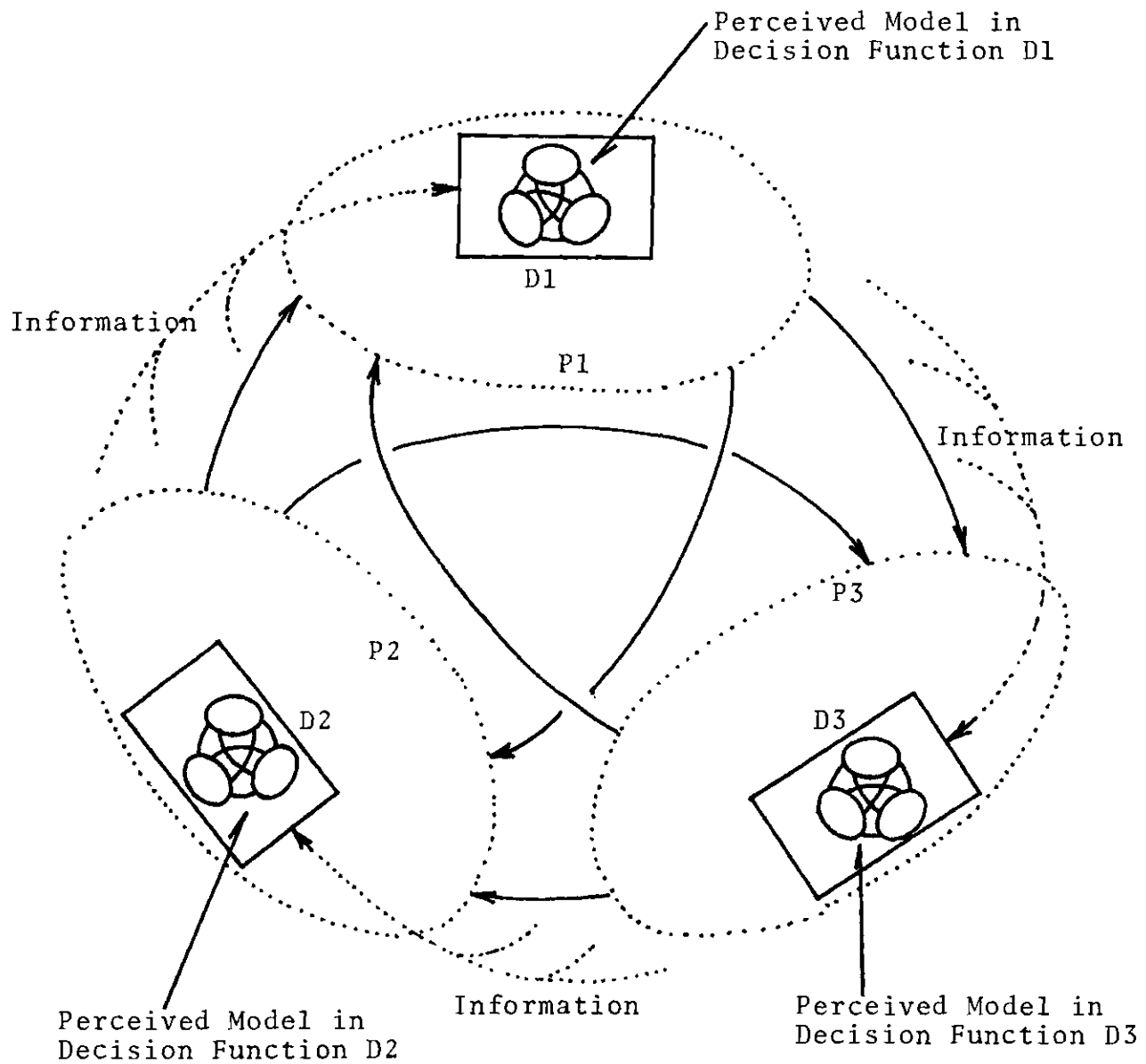


Figure 3-7. Perceived Model in a System

$$\Delta Sp2 = 1 \times B12 \times A12 \quad (III-6)$$

The amount of change $\Delta Sp1$ which is produced by $\Delta Sp2$ is given as

$$\Delta Sp1 = |\Delta Sp2| \times K21 \times B21 \times A21 \quad (III-7)$$

Therefore,

$$\Delta Sp1 = 1 \times B12 \times |A12| \times K21 \times B21 \times A21 \quad (III-8)$$

This equation explains the gain G of $K12$ to $Sp1$ through loop $P1-P2-P1$; then,

$$G12(|2|) = B12 \times |A12| \times K21 \times B21 \times A21 \quad (III-9)$$

Gain $G12$ of effort for $X12$ to $S1$ through the loop $P1-P2-P3-P1$ is given as follows:

$$G12(1231) = B12 \times |A12| \times K23 \times B23 \times |A23| \quad (III-10) \\ \times K31 \times B31 \times A31$$

Total gain $G12$ for $X12$ is:

$$G12 = G12(121) + G12(1231) \quad (III-11)$$

The value of the gain is just the system's estimated contribution to the participant's Situation Function.

When we consider the real problem, it is reasonable to assume the attitude of the decision maker as follows: it is easier for the decision maker to consider the simple loop than the complicated loop; therefore, as the general tendency, the decision maker will observe and consider the simple loop much more than the complicated loop and will ignore the complicated loop because the difficulty of consideration.

Then the Consideration Coefficient W_{ab} will affect the value of the gain, depending on the complexity of the loop, as follows:

$$G_{12} = W_{12}(121) \times G_{12}(121) + W_{12}(1231) \times G_{12}(1231) \quad (\text{III-12})$$

As an example, a value can be given to $W_{12}(121)$ and $W_{12}(1231)$ as follows:

$$W_{12}(121) = 1.0 \quad (\text{III-13})$$

$$W_{12}(1231) = 0.8 \quad (\text{III-14})$$

Then,

$$G_{12} = (1.0) \times G_{12}(121) + (0.8) \times G_{12}(1231) \quad (\text{III-15})$$

The value of G_{12} is the system's estimated contribution to the Situation Function for the action X_{12} .

In the same way, G_{13} will be determined as follows:

$$G_{13} = W_{13}(131) \times G_{13}(131) + W_{13}(1321) \times G_{13}(1321) \quad (\text{III-16})$$

In the simplest way, K_{12} , K_{13} are determined in proportion to the value of G_{12} , G_{13} as follows:

$$K_{12} = \frac{G_{12}}{G_{12} + G_{13}} \quad K_{13} = \frac{G_{13}}{G_{12} + G_{13}} \quad (\text{III-17})$$

The Distribution Coefficients K_{12} , K_{13} are the outputs from the Decision Function. Thus, the actual action will be determined by these Distribution Coefficients K_{12} , K_{13} in the Action Function as mentioned in the earlier section of the chapter.

When the manner of calculating the gain was discussed, the observed value of K was used for estimating the system's reaction. This method is based on an assumption that the value of K does not change or changes very slowly. If the behavior of the whole system is very slow, this way is justified. On the other hand, most real situations are not of this type.

There are many ways to assume the value of K :

1. Taking the observed value of K .

2. Assuming the value of K based on a value of Need Coefficient A.
3. Forecasting the value of K based on a present tendency.

The method of determining K depends on the characteristics of the participants.

The second method can be explained as follows.

The decision making of the participant should not be based on the gain resulting from the present action, but on the gain resulting from the future action which he will take. Thus, it is wrong to use the observed value of the Distribution Coefficient of the other participant. In Figure 3-6, participant P1 is deciding his action X12. If the Need Coefficient A12 is large, P1 can require P2 to contribute to increase the value of P1's Situation Function, that is, P1 can expect the value of K23, K21 to be in proportion to the value of A12.

Then the expected value of each K is:

$$K_{23} = K_{21} = \frac{A_{12}}{A_{12} + A_{32}} \quad (\text{III-18})$$

The third method of assuming the value of K can be explained as follows. One participant can assume that the other participant will continue to experience the current rate of change of his Distribution Coefficient. For example, if the historical curve of participant A's Distribution

Coefficient is as shown in Figure 3-8, the other participants B, C, D, will assume that this tendency will continue.

Procedure for Determining the Need Coefficient
and Transformation Coefficient

For the application of this modeling method to the real problem, the actual numerical value must be given to each Need Coefficient and Transformation Coefficient. For this purpose, at first, the dimension of each action will be determined arbitrarily so that each action will be defined clearly, and increases or decreases in the action will be understandable conceptually. This is especially necessary for the non-physical action, for example, political support, hospitality, psychological help.

After the determination of the dimension of the action, the real number of the Need Coefficient will be derived in the same way as the determination of utility function proceeds. That is, the value of each coefficient must be determined so that the value multiplied by both one coefficient and one unit action is comparable to the value multiplied by both another coefficient and another unit action. For example, participant Pa is receiving two kinds of reaction Xba, Xca from the system and he has Need Coefficient Aba, Aca for each reaction Xba, Xca. Then, if the value of $(Aba) \times (\text{unit of } Xba)$ equals the value of $(Aca) \times (\text{unit of } Xca)$ the ratio of the values of these Need Coefficients is

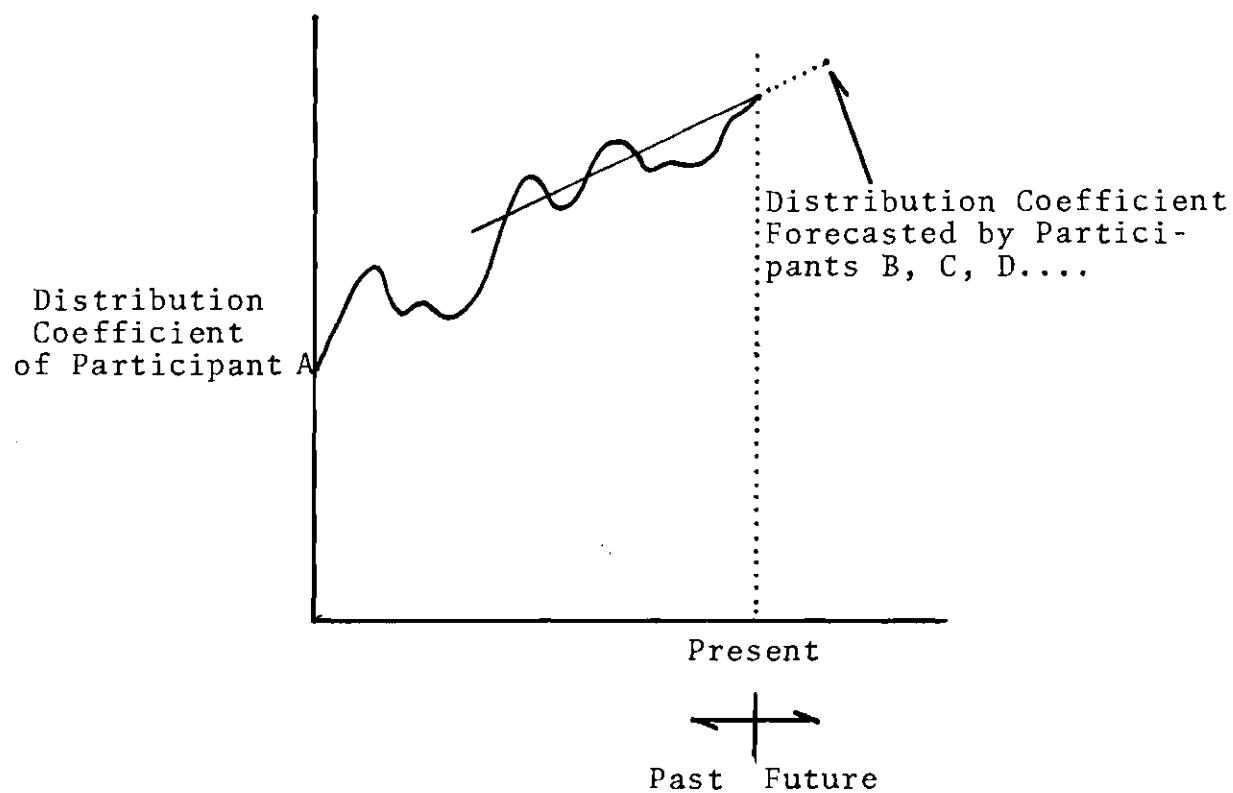


Figure 3-8. Forecasted Distribution Coefficient

appropriate. As the actual method, proposals with different Need Coefficients will be given to the decision maker as follows:

Question: "Which do you prefer?"

Proposal A. $(1.0) \times (\text{unit action of } X_{ba})$

Proposal B. $(1.0) \times (\text{unit action of } X_{ca})$

Then, if the participant chooses proposal A, a different value of the Need Coefficient must be chosen for the proposal; then, the same type of question will be given to the decision maker again, as follows:

A. $(0.8) \times (\text{unit action of } X_{ba})$

B. $(1.0) \times (\text{unit action of } X_{ca})$

These steps are repeated until the decision maker reaches the point where he cannot choose one proposal over the other proposal; at that time, the two proposals can be considered comparable and the ratio of the value of these Need Coefficients is appropriate.

After the determination of the Need Coefficient, the Transformation Coefficient will be determined as follows:

By assuming the maximum value of each action from the system, the maximum value of the motivation of one participant is determined and, therefore, the maximum value of his reaction. Then, by assuming that this reaction occurs entirely as one type of action, such as political action, the transformation coefficient can be deduced from the amount of that type of behavior. This process is repeated

for each action, respectively. Further calibration can be achieved for beginning with the assumption of less than maximum action from the system.

The Need Coefficient in the Situation Function of the participant is defined by the other variable or function which consists of many variables or simple time functions or the like. The form in which the Need Coefficient is defined depends on the problem. Some examples of the form are introduced below.

A. Feedback Loop From the Action to the Need Coefficient

Here, the accumulated value of the action decreases the value of the Need Coefficient. For example, one participant can teach a specific language to the other, in turn, he wants to learn another language from the second participant. The language proficiency of the participant is explained as the accumulated value of the action, and generally, an increase in language proficiency decreases the value of the Need Coefficient.

B. Feedback Loop by the Ratio of Two Actions to the Need Coefficient

To keep the balance of the relationship, the ratio of two accumulated actions affects the Need Coefficient of one of the actions. For example, in Figure 3-9, if the nation A is increasing some kind of action, XAC, military support, political support, nation B will increase the value of the Need Coefficient for the action XAB in order to keep the

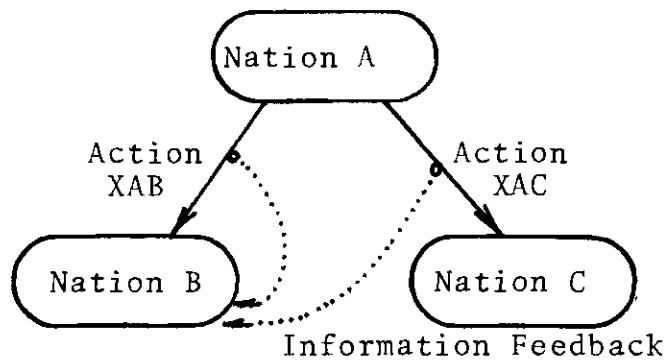


Figure 3-9. Feedback Loop by the Ratio of Two Actions

balance of the relationship among the nations.

C. Time Function Determining the Need Coefficient

Some Need Coefficients will be determined by forces outside the system. For example, assume a nation imports some commodity from another nation. If the need for the commodity depends on the GNP of the nation, the Need Coefficient can be represented by the time function of GNP, or simply a time function.

Use of the examples which were mentioned above depends on the problem. Obviously, skillful observation of the real problem is required to make these relationships clear.

CHAPTER IV

SIMPLE EXAMPLE OF THE PROCEDURE

In this chapter, a simple model in which three persons are participating in a reciprocal language learning situation is discussed. The purpose of using this example is to demonstrate the ability of the modeling technique to describe and explain some typical behavior patterns.

Each person has the following available action:

Mr. A: teaching English

Mr. B: teaching German

Mr. C: teaching French

and each person has the following Need Coefficient for the reaction:

Mr. A: comparatively large need for proficiency in German; comparatively small need for proficiency in French

Mr. B: Comparatively large need for proficiency in French; comparatively small need for proficiency in English

Mr. C: comparatively large need for proficiency in English; comparatively small need for proficiency in German

The relationship between these three persons is

illustrated in Figure 4-1.

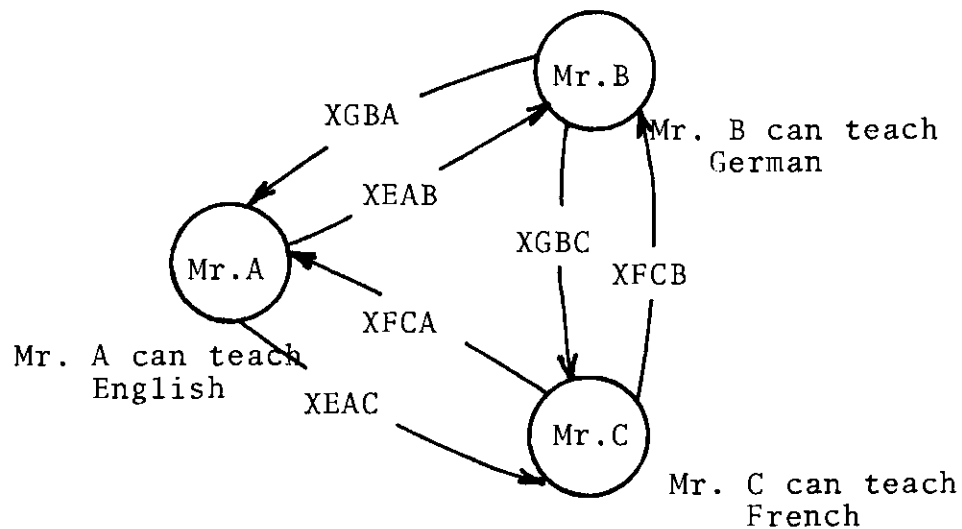


Figure 4-1. Relationship of Three Participants in Reciprocal Language Learning Model

Modeling

In the above figure, each variable for the actions is defined as follows:

XEAB: English teaching action by Mr. A to Mr. B

XEAC: English teaching action by Mr. A to Mr. C

XGBA: German teaching action by Mr. B to Mr. A

XGBC: German teaching action by Mr. B to Mr. C

XFCA: French teaching action by Mr. C to Mr. A

XFCB: French teaching action by Mr. C to Mr. B

The observed value of action QXEAB is given as follows:

$$QXEAB.K = QXEAB.J + (DT)(1/DEL)(XEAB.JK - QXEAB.J) \quad (IV-1)$$

In this equation,

QXEAB: observed value of English teaching by Mr. A
to Mr. B

DT: unit time - 1 week

DEL: delay time - 1 week

In the same way, the observed values of all the
remaining actions are given below.

$QXEAC.K = QXEAC.J + (DT)(1/DEL)(XEAC.JK - QXEAC.J)$ (IV-2)

QXEAC: Observed value of English teaching by Mr. A
to Mr. C

$QXGBA.K = QXGBA.J + (DT)(1/DEL)(XGBA.JK - QXGBA.J)$ (IV-3)

QXGBA: observed value of German teaching by Mr. B
to Mr. A

$QXGBC.K = QXGBC.J + (DT)(1/DEL)(XGBC.JK - QXGBC.J)$ (IV-4)

QXGBC: observed value of German teaching by Mr. B
to Mr. C

$QXFCA.K = QXFCA.J + (DT)(1/DEL)(XFCA.JK - QXFCA.J)$ (IV-5)

QXFCA: observed value of French teaching by Mr. C
to Mr. A

$QXFCB.K = QXFCB.J + (DT)(1/DEL)(XFCB.JK - QXFCB.J)$ (IV-6)

QXFCB: observed value of French teaching by Mr. C
to Mr. B

In this example, it is assumed that each Situation
Function is an additive function.

$$\begin{aligned} \text{Situation Function} = & (\text{Need Coefficient}) \times (\text{Action \#1}) \\ & + (\text{Need Coefficient}) \times (\text{Action \#2}) \quad (\text{IV-7}) \end{aligned}$$

By applying this formula, each Situation Function is expressed as follows:

$$\text{SA.K} = (\text{AGBA.K}) (\text{QXGBA.K}) + (\text{AFCA.K}) (\text{QXFCA.K}) \quad (\text{IV-8})$$

SA: situation of Mr. A

AGBA: Need Coefficient of Mr. A for German teaching action by Mr. B

AFCA: Need Coefficient of Mr. A for French teaching action by Mr. C

In this example, the Need Coefficient is defined by the function of language proficiency, that is, the Need Coefficient for the language will decrease as proficiency in that language increases. This concept will be discussed in detail in a later part of this chapter.

$$\text{SB.K} = (\text{AEAB.K}) (\text{QXEAB.K}) + (\text{AFCB.K}) (\text{QXFCB.K}) \quad (\text{IV-9})$$

SB: situation of Mr. B

AEAB: Need Coefficient of Mr. B for English teaching action by Mr. A

AFCB: Need Coefficient of Mr. B for French teaching action by Mr. C

$$\text{SC.K} = (\text{AGBC.K}) (\text{QXGBC.K}) + (\text{AEAC.K}) (\text{QXEAC.K}) \quad (\text{IV-10})$$

SC: situation of Mr. C

AGBC: Need Coefficient of Mr. C for German teaching

action by Mr. B

AEAC: Need Coefficient of Mr. C for English teaching

action by Mr. A

In this example, it is assumed that the Motivation Function is defined as follows:

$$MA.K = SA.K + 1.0 \quad (IV-11)$$

MA: Motivation of Mr. A

$$MB.K = SB.K + 1.0 \quad (IV-12)$$

MB: Motivation of Mr. B

$$MC.K = SC.K + 1.0 \quad (IV-13)$$

MC: Motivation of Mr. C

The constant value one (1.0) in the above Motivation Function means that each participant has the motivation to exert at least one hour of teaching action per week even if there is no action from the other participants.

The Action Function, which determines each action, is given by the following function:

$$\text{Action} = (\text{Motivation})(\text{Distribution Coefficient})(\text{Transformation Coefficient}) \quad (IV-14)$$

By applying this formula to this example, each Action Function is derived as follows:

Action Function of Mr. A

$$XEAB.KL = (MA.K) (KXEAB.K) (BXEAB) \quad (IV-15)$$

MA: motivation of Mr. A

KXEAB: Distribution Coefficient of English teaching
action to Mr. B

BXEAB: Transformation Coefficient for English
teaching action to Mr. B

$$XEAC.KL = (MA.K) (KXEAC.K) (BXEAC) \quad (IV-16)$$

MA: motivation of Mr. A

KXEAC: Distribution Coefficient of English teaching
action to Mr. C

BXEAC: Transformation Coefficient for English
teaching action to Mr. C

Action Function of Mr. B

$$XGBA.KL = (MB.K) (KXGBA.K) (BXGBA) \quad (IV-17)$$

MB: motivation of Mr. B

KXGBA: Distribution Coefficient of German teaching
action to Mr. A

BXGBA: Transformation Coefficient for German
teaching action to Mr. A

$$XGBC.KL = (MB.K) (KXGBC.K) (BXGBC) \quad (IV-18)$$

MB: motivation of Mr. B

KXGBC: Distribution Coefficient of German teaching
action to Mr. C

BXGBC: Transformation Coefficient for German
teaching action to Mr. C

Action Function of Mr. C

$$XFCA.KL = (MC.K) (KXFCA.K) (BXFCA) \quad (IV-19)$$

MC: motivation of Mr. C

KXFCA: Distribution Coefficient of French teaching action to Mr. A

BXFCA: Transformation Coefficient for French teaching action to Mr. A

$$XFCB.KL = (MC.K) (KXFCB.K) (BXFCB) \quad (IV-20)$$

MC: motivation of Mr. C

KXFCB: Distribution Coefficient of French teaching action to Mr. B

BXFCB: Transformation Coefficient for French teaching action to Mr. B

Gain Equation for Mr. A

First, the loop Mr. A-Mr. B-Mr. A is considered in order to calculate the gain from effort for the action XEAB, Mr. A teaching English to Mr. B.

In Figure 4-2, the amount of change in the value of the Situation Function of participant Mr. B which is produced by one unit of effort for XEAB is as follows:

$$\Delta SB = 1 \times BXEAB \times AEAB \quad (IV-21)$$

and also

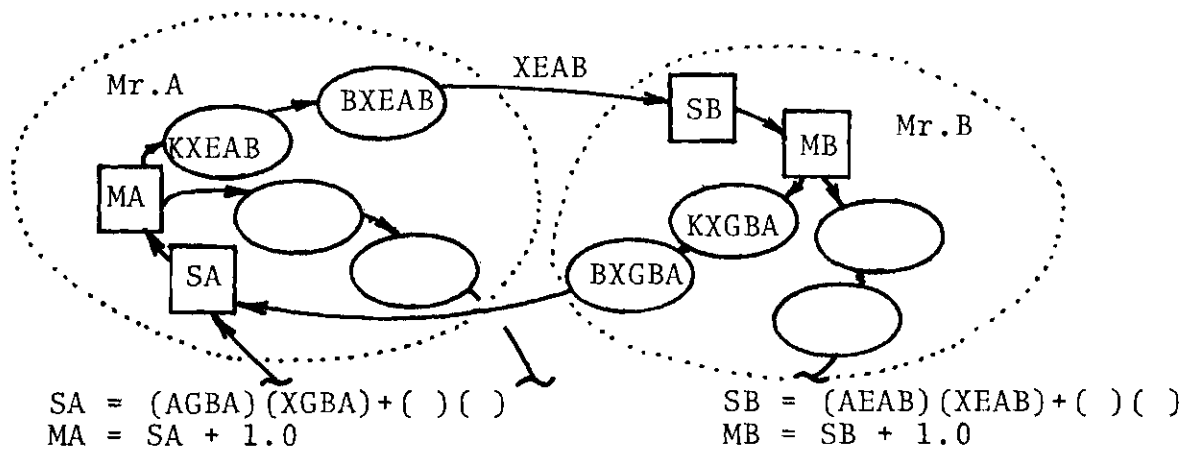


Figure 4-2. Two-Step Loop Considered for Gain Calculation of XEAB

$$\Delta MB = |\Delta SB| \quad (IV-22)$$

BXEAB: Transformation Coefficient for English teaching action by Mr. B

AEAB: Need Coefficient of Mr. B for English teaching action by Mr. A

The amount of change SA which is produced by ΔMB is given as

$$\Delta SA = \Delta MB \times KXGBA \times BXGBA \times AGBA \quad (IV-23)$$

Therefore,

$$\Delta SA = 1 \times BXEAB \times |AEAB| \times KXGBA \times BXGBA \times AGBA \quad (IV-24)$$

This equation is the gain of KXEAB to SA through loop Mr. A-Mr. B-Mr. A. Then,

$$DAEAB.K = (BXEAB) | (AEAB.K) | (KXGBA.K) (BXGBA) (AGBA.K) \quad (IV-25)$$

In the same way, the gain from effort for the action XEAB through the loop Mr. A-Mr. B-Mr. C-Mr. A is given as follows (see Figure 4-3):

$$DBEAB.K = (BXEAB) | (AEAB.K) | (KXGBC.K) (BXGBC) | (AGBC.K) | (KXFCA.K) (BXFCA) (AFCA.K) \quad (IV-26)$$

$$SA = () () + (AFCA) (XFCA) \\ MA = SA + 1.0$$

$$SB = (AEAB) (XEAB) + () () \\ MB = SB + 1.0$$

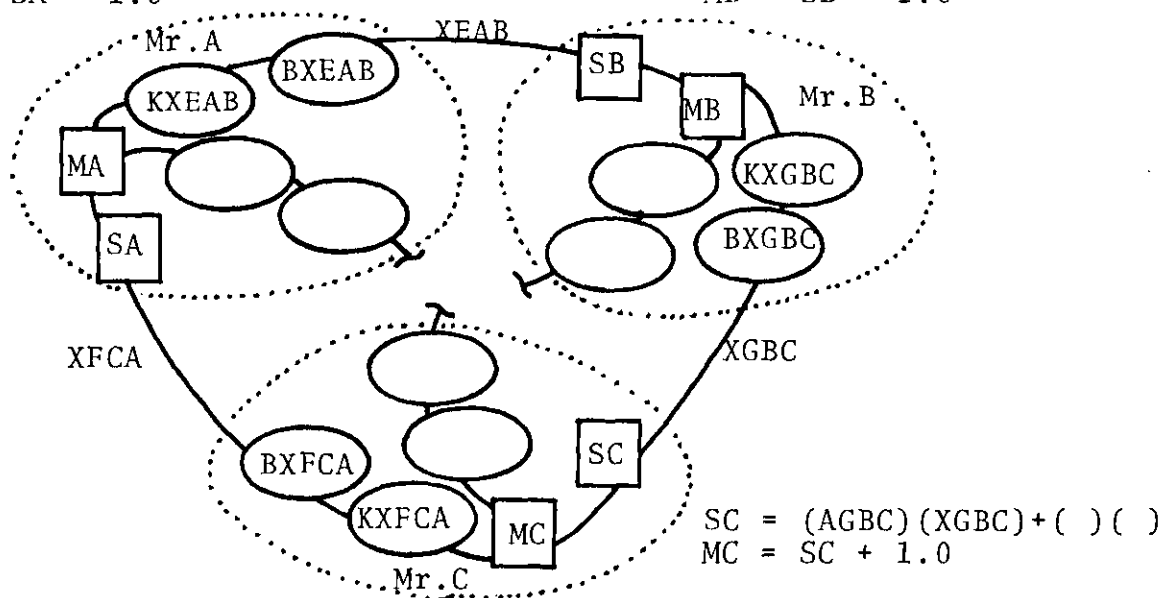


Figure 4-3. Three-Step Loop Considered for Gain Calculation of XEAB

By giving the value of the Consideration Coefficient for each loop, the total gain from effort for XEAB is determined as follows:

$$DTEAB.K = (DAEAB.K)(1.0) + (DBEAB.K)(0.8) \quad (IV-27)$$

In this example, it is assumed that only two-way and three-way loops are considered for calculation of gain. The different values of the Consideration Coefficients in the above gain equation mean that the participant is considering the simple two-step loop more important than the three-step loop in estimating system reaction.

By the same procedure, the gain for each of the other actions by each person is determined as follows:

Gain from effort for XEAC, Mr. A teaching English to Mr. C

(IV-28)

$$DAEAC.K = (BXEAC) \mid (AEAC.K) \mid (KXFCA.K)(BXFCA)(AFCA.K)$$

$$DBEAC.K = (BXEAC) \mid (AEAC.K) \mid (KXFCB.K)(BXFCB) \mid (AFCB.K) \mid (KXGBA.K)(BXGBA)(AGBA.K) \quad (IV-29)$$

$$DTEAC.K = (DAEAC.K)(1.0) + (DBEAC.K)(0.8) \quad (IV-30)$$

DAEAC: Gain of KXEAC to SA through the loop
Mr. A-Mr. C-Mr. A

DBEAC: Gain of KXEAC to SA through the loop
Mr. A-Mr. C-Mr. B-Mr. A.

DTEAC: Total gain for KXEAC to SA.

Gain Equation for Mr. B

Gain from effort for XGBA, Mr. B teaching German to

Mr. A

(IV-31)

$$\text{DAGBA.K} = (\text{BXGBA}) \mid (\text{AGBA.K}) \mid (\text{KXEAB.K}) (\text{BXEAB}) (\text{AEAB.K})$$

$$\text{DBGBA.K} = (\text{BXGBA}) \mid (\text{AGBA.K}) \mid (\text{DKEAC.K}) (\text{BXEAC}) \mid (\text{AEAC.K}) \mid (\text{KXFCB.K}) (\text{BXFCB}) (\text{AFCB.K}) \quad (\text{IV-32})$$

$$\text{DTGBA.K} = (\text{DAGBA.K}) (1.0) + (\text{DBGBA.K}) (0.8) \quad (\text{IV-33})$$

DAGBA: Gain of KXGBA to SB through the loop

Mr. B-Mr. A-Mr. B

DBGBA: Gain of KXGBA to SB through the loop

Mr. B-Mr. A-Mr. C-Mr. B.

DTGBA: Total gain for KXGBA to SB.

Gain from effort for XGBC, Mr. B teaching German to

Mr. C

(IV-34)

$$\text{DAGBC.K} = (\text{BXGBC}) \mid (\text{AGBC.K}) \mid (\text{KXFCB.K}) (\text{BXFCB}) (\text{AFCB.K})$$

$$\text{DBGBC.K} = (\text{BXGBC}) \mid (\text{AGBC.K}) \mid (\text{KXFCA.K}) (\text{BXFCA}) \mid (\text{AFCA.K}) \mid (\text{KXEAB.K}) (\text{BXEAB}) (\text{AEAB.K}) \quad (\text{IV-35})$$

$$\text{DTGBC.K} = (\text{DAGBC.K}) (1.0) + (\text{DBGBC.K}) (0.8) \quad (\text{IV-36})$$

DAGBC: Gain of KXGBC to SB through the loop

Mr. B-Mr. C-Mr. B

DBGBC: Gain of KXGBC to SB through the loop

Mr. B-Mr. C-Mr. A-Mr. B

DTGBC: Total gain for KXGBC to SB

Gain Equation for Mr. C

Gain of effort for XFCA, Mr. C teaching French to Mr. A
(IV-37)

$$DAFCA.K = (BXFCA) | (AFCA.K) | (KXEAC.K) (BXEAC) (AFEAC.K)$$

$$DBFCA.K = (BXFCA) | (AFCA.K) | (KXEAB.K) (BXEAB) | (AEAB.K) | \\ (KXGBC.K) (BXGBC) (AGBC.K) \quad (IV-38)$$

$$DTFCA.K = (DAFCA.K) (1.0) + (DBFCA.K) (0.8) \quad (IV-39)$$

DAFCA: Gain of KXFCA to SA through the loop
Mr. C-Mr. A-Mr. C.

DBFCA: Gain of KXFCA to SC through the loop
Mr. C-Mr. A-Mr. B-Mr. C

DTFCA: Total gain for KXFCA to SC

Gain from effort for XFCB, Mr. C teaching French to Mr. B
(IV-40)

$$DAFCB.K = (BXFCB) | (AFCB.K) | (KXGBC.K) (BXGBC) (AGBC.K)$$

$$DBFCB.K = (BXFCB) | (AFCB.K) | (KXGBA.K) (BXGBA) | (AGBA.K) | \\ (KXEAC.K) (BXEAC) (AEAC.K) \quad (IV-41)$$

$$DTFCB.K = (DAFCB.K) (1.0) + (DBFCB.K) (0.8) \quad (IV-42)$$

DAFCB: Gain of KXFCE to SC through the loop
Mr. C-Mr. B-Mr. C.

DBFCB: Gain of KXFCE to SC through the loop
Mr. C-Mr. B-Mr. A-Mr. C

DTFCB: Total gain for KXFCE to SC.

In this example, as the simplest way, each Distribution Coefficient is determined in proportion to the value of each gain.

Distribution Coefficient of Mr. A

$$KXEAB.K = DTEAB.K / (DTEAB.K + DTEAC.K) \quad (IV-43)$$

$$KXEAC.K = DTEAC.K / (DTEAB.K + DTEAC.K) \quad (IV-44)$$

KXEAB: Distribution Coefficient of English teaching
action to Mr. B

KXEAC: Distribution Coefficient of English teaching
action to Mr. C

Distribution Coefficient of Mr. B

$$KXGBA.K = DTGBA.K / (DTGBA.K + DTGBC.K) \quad (IV-45)$$

$$KXGBC.K = DTGBC.K / (DTGBA.K + DTGBC.K) \quad (IV-46)$$

KXGBA: Distribution Coefficient of German teaching
action to Mr. A

KXGBC: Distribution Coefficient of German teaching
action to Mr. C

Distribution Coefficient of Mr. C

$$KXFCA.K = DTFCA.K / (DTFCA.K + DTFCB.K) \quad (IV-47)$$

$$KXFBC.K = DTFCB.K / (DTFCA.K + DTFCB.K) \quad (IV-48)$$

KXFCA: Distribution Coefficient of French teaching
action to Mr. A

KXFBC: Distribution Coefficient of French teaching
action to Mr. B

As mentioned in Chapter III, there are many ways to assume the value of the Distribution Coefficient for calculation of gain. The application of each method to this problem will be discussed as follows:

Taking the Observed Value of the Distribution Coefficient
for Calculation of Gain

The observed value of the Distribution Coefficient
is given by the following equation: (IV-49)

$$\text{AYEAB.K} = \text{AYEAB.J} + (\text{DT})(1/\text{DEL } 1)(\text{KXEAB.J} - \text{AYEAB.J})$$

$$\text{DEL } 1 = 3.0 \quad (\text{IV-50})$$

AYEAB: Observed value of Distribution Coefficient
for XEAB

KXEAB: Distribution Coefficient for XEAB
(IV-51)

$$\text{AYEAC.K} = \text{AYEAC.J} + (\text{DT})(1/\text{DEL } 1)(\text{KXEAC.J} - \text{AYEAC.J})$$

AYEAC: Observed value of Distribution Coefficient
for XEAC

KXEAC: Distribution Coefficient for XEAC
(IV-52)

$$\text{AYGBA.K} = \text{AYGBA.J} + (\text{DT})(1/\text{DEL } 1)(\text{KXGBA.J} - \text{AYGBA.J})$$

AYGBA: Observed value of Distribution Coefficient
for XGBA

KXGBA: Distribution Coefficient for XGBA
(IV-53)

$$\text{AYGBC.K} = \text{AYGBC.J} + (\text{DT})(1/\text{DEL } 1)(\text{KXGBC.J} - \text{AYGBC.J})$$

AYGBC: Observed value of Distribution Coefficient
for XGBC

KXGBC: Distribution Coefficient for XGBC
(IV-54)

$$\text{AYFCA.K} = \text{AYFCA.J} + (\text{DT})(1/\text{DEL } 1)(\text{KXFCA.J} - \text{AYFCA.J})$$

AYFCA: Observed value of Distribution Coefficient
for XFCA

KXFCA: Distribution Coefficient for XFCA

$$AYFCB.K = AYFCB.J + (DT)(1/DEL 1)(KXFCB.J - AYFCB.J) \quad (IV-55)$$

AYFCB: Observed value of Distribution Coefficient
for XFCB

KXFCB: Distribution Coefficient for XFCB

Assuming the Value of the Distribution Coefficient Based on
a Value of the Need Coefficient

By applying the equation III-13, to this problem, the
following equation is derived:

$$AYEAB.K = AEAB.K / (AEAB.K + AECB.K) \quad (IV-56)$$

AYEAB: Assumed Distribution Coefficient for XEAB

AEAB: Need Coefficient for XEAB

AFCB: Need Coefficient for XFCB

$$AYEAC.K = AEAC.K / (AEAC.K + AGBC.K) \quad (IV-57)$$

AYEAC: Assumed Distribution Coefficient for XEAC

AEAC: Need Coefficient for XEAC

AGBC: Need Coefficient for XGBC

$$AYGBA.K = AGBA.K / (AGBA.K + AFCA.K) \quad (IV-58)$$

AYGBA: Assumed Distribution Coefficient for XGBA

AGBA: Need Coefficient for XEBA

AFCA: Need Coefficient for XFCA

$$AYGBC.K = AGBC.K / (AGBC.K + AEAC.K) \quad (IV-59)$$

AYGBC: Assumed Distribution Coefficient for XGBC

AGBC: Need Coefficient for XGBC

AEAC: Need Coefficient for XEAC

$$AYFCA.K = AFCA.K / (AGBA.K + AFCA.K) \quad (IV-60)$$

AYFCA: Assumed Distribution Coefficient for XFCA

AFCA: Need Coefficient for XFCA

AGBA: Need Coefficient for XGBA

$$AYFCB.K = AFCB.K / (AEAB.K + AFCB.K) \quad (IV-61)$$

AYFCB: Assumed Distribution Coefficient for XFCB

AFCB: Need Coefficient for XFCB

AEAB: Need Coefficient for XEAB

Forecasting the Value of the Distribution Coefficient Based
on a Present Tendency

This method is given by the following equation: (IV-62)

$$AHEAB.K = AHEAB.J + (DT)(1/DEL\ 2)(KXEAB.J - AHEAB.J) \quad (IV-63)$$

$$BHEAB.K = BHEAB.J + (DT)(1/DEL\ 3)(KXEAB.J - BHEAB.J)$$

$$SHEAB.K = (BHEAB.K - AHEAB.K)(0.25) \quad (IV-64)$$

$$AYEAB.K = KXEAB.J + SHEAB.K \quad (IV-65)$$

$$DEL\ 2 = 6 \quad (IV-66)$$

$$DEL\ 3 = 2 \quad (IV-67)$$

AHEAB: Average delay information 6 time periods

BHEAB: Average delay information 2 time periods

SHEAB: Average change per time period in the value
of the Distribution Coefficient for action
XEAB during 4 units of time

AYEAB: Forecasted Distribution Coefficient for
action XEAB

$$\text{AHEAC.K} = \text{AHEAC.J} + (\text{DT})(1/\text{DEL } 2)(\text{KXEAC.J} - \text{AHEAC.J}) \quad (\text{IV-68})$$

$$\text{BHEAC.K} = \text{BHEAC.J} + (\text{DT})(1/\text{DEL } 3)(\text{KXEAC.J} - \text{BHEAC.J}) \quad (\text{IV-69})$$

$$\text{SHEAC.K} = (\text{BHEAC.K} - \text{AHEAC.K})(0.25) \quad (\text{IV-70})$$

$$\text{AYEAC.K} = \text{KXEAC.J} + \text{SHEAC.K} \quad (\text{IV-71})$$

SHEAC: Average change per time period in the value
of the Distribution Coefficient for action
XEAC during 4 units of time

AYEAC: Forecasted Distribution Coefficient for action
XEAC

$$\text{AHGBA.K} = \text{AHGBA.J} + (\text{DT})(1/\text{DEL } 2)(\text{KXGBA.J} - \text{AHGBA.J}) \quad (\text{IV-72})$$

$$\text{BHGBA.K} = \text{BHGBA.J} + (\text{DT})(1/\text{DEL } 3)(\text{KXGBA.J} - \text{BHGBA.J}) \quad (\text{IV-73})$$

$$\text{SHGBA.K} = (\text{BHGBA.K} - \text{AHGBA.K})(0.25) \quad (\text{IV-74})$$

$$\text{AYGBA.K} = \text{KXGBA.J} + \text{SHGBA.K} \quad (\text{IV-75})$$

SHGBA: Average change per time period in the value
of the Distribution Coefficient for the
action XGBA during 4 units of time

AYGBA: Forecasted Distribution Coefficient for
action XGBA

$$\text{AHGBC.K} = \text{AHGBC.J} + (\text{DT})(1/\text{DEL } 2)(\text{KXGBC.J} - \text{AHGBC.J}) \quad (\text{IV-76})$$

$$\text{BHGBC.K} = \text{BHGBC.K} + (\text{DT})(1/\text{DEL } 3)(\text{KXGBC.J} - \text{BHGBC.J}) \quad (\text{IV-77})$$

$$\text{SHGBC.K} = (\text{BHGBC.K} - \text{AHGBC.K})(0.25) \quad (\text{IV-78})$$

$$\text{AYGBC.K} = \text{KXGBC.J} - \text{SHGBC.K} \quad (\text{IV-79})$$

SHGBC: Average change per time period in the value
of the Distribution Coefficient for the action
XGBC during 4 units of time

AYGBC: Forecasted Distribution Coefficient for the
action XGBC.

$$\text{AHFCA.K} = \text{AHFCA.J} + (\text{DT})(1/\text{DEL } 2)(\text{KXFCA.J} - \text{AHFCA.J}) \quad (\text{IV-80})$$

$$\text{BHFC A.K} = \text{BHFC A.J} + (\text{DT})(1/\text{DEL } 3)(\text{KXFCA.J} - \text{BHFC A.J}) \quad (\text{IV-81})$$

$$\text{SHFCA.K} = (\text{BHFC A.K} - \text{AHFCA.K})(0.25) \quad (\text{IV-82})$$

$$\text{AYFCA.K} = \text{KXFCA.J} + \text{SHFCA.K} \quad (\text{IV-83})$$

SHFCA: Average change per time period in the value
of the Distribution Coefficient for the
action SFCA during 4 units of time

AYFCA: Forecasted Distribution Coefficient for the
action XFCA

$$\text{AHFCB.K} = \text{AHFCB.J} + (\text{DT})(1/\text{DEL } 2)(\text{KXFCB.J} - \text{AHFCB.J}) \quad (\text{IV-84})$$

$$\text{BHFCB.K} = \text{BHFCB.J} + (\text{DT})(1/\text{DEL } 3)(\text{KXFCB.J} - \text{BHFCB.J}) \quad (\text{IV-85})$$

$$\text{SHFCB.K} = (\text{BHFCB.K} - \text{AHFCB.K})(0.25) \quad (\text{IV-86})$$

$$\text{AYFCB.K} = \text{KXFCB.J} + \text{SHFCB.K} \quad (\text{IV-87})$$

SHFCB: Average change per time period in the value
of the Distribution Coefficient for the
action XFCB during 4 units of time

AYFCB: Forecasted Distribution Coefficient for the
action XFCB.

Simulation of the Model

Simulation of the model for different cases was
performed as follows:

(1) Assumed Distribution Coefficient (ADC)*

Three methods of assuming the value of the Distribution Coefficient for calculation of gain were used.

(a) Assuming the value of the Distribution Coefficient based on a value of the Need Coefficient (N/C)* (refer to Chapter III).

(b) Using the observed value of the Distribution Coefficient (O/V)* (refer to Chapter III).

(c) Forecasting the value of the Distribution Coefficient based on a present tendency (F/V)* (refer to Chapter III).

(2) Need Coefficient Function (NF)*

The Need Coefficient for learning a language is defined by the function of language proficiency, which is defined as the accumulated learning hours minus decrease of proficiency by forgetting. Each Need Coefficient is determined by the following equation:

$$SXGBA.K = SXGBA.J + (DT)(XGBA.JK - LGBA.JK) \quad (IV-88)$$

$$LGBA.KL = (SXGBA.K)(0.01) \quad (IV-89)$$

$$AGBAO.K = A + (SXGBA.K)(B) \quad (IV-90)$$

$$AGBA.K = \text{MAX}(0.0, AGBAO.K) \quad (IV-91)$$

SXGBA: German language proficiency of Mr. A

LGBA: Decrease of proficiency by forgetting (1% of proficiency)

The abbreviations marked with an asterisk () are used in the Simulation Run List, Figure 4-11.

AGBAO: Original Need Coefficient determined by the
function of German language proficiency

AGBA: Need Coefficient for German learning by Mr. A

Other Need Coefficient Functions for AFCA, AEAB, AFCEB, AGBC, AEAC, are determined in the same way. By changing the value of A and B in the function as follows, two different cases were simulated.

Need Coefficient	(A) Different Need/C (2,1)*		(B) Same Need/C (1,1)*	
	A	B	A	B
AGBA	1.0	-0.002	1.0	-0.002
AFCA	2.0	-0.004	1.0	-0.002
AEAB	2.0	-0.004	1.0	-0.002
AFCEB	1.0	-0.002	1.0	-0.002
AGBC	2.0	-0.004	1.0	-0.002
AEAC	1.0	-0.002	1.0	-0.002

Figure 4-4. Two Cases of Simulation in Different Need Coefficient Functions

(a) Different Need Coefficient (2,1)*. The relationship between Need Coefficient and proficiency is described as follows:

(i) Need Coefficient Function with A = 2.0, B = -0.004

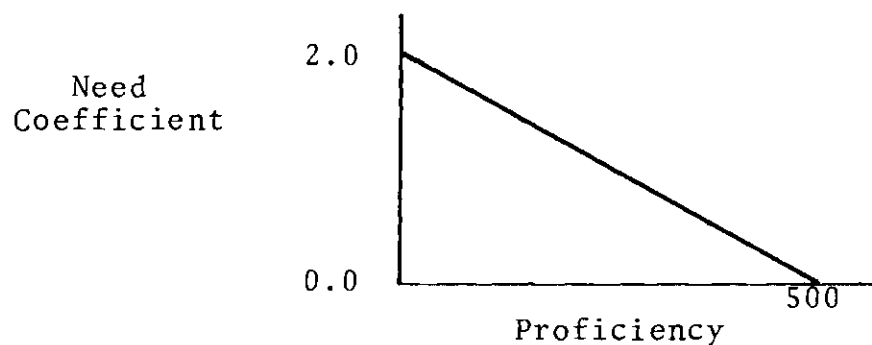


Figure 4-5. Need Coefficient Function Type A

(ii) Need Coefficient Function with $A = 1.0$,
 $B = -0.002$

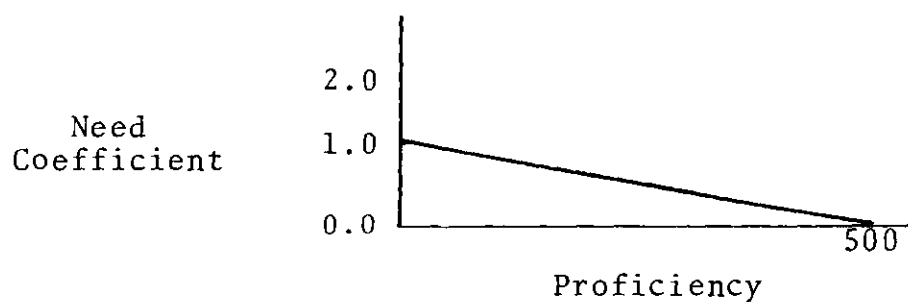


Figure 4-6. Need Coefficient Function Type B

Each participant has each Need Function as follows:

L: Relative large value of Need Coefficient for the action by Need Coefficient Function with $A = 2.0$, $B = -0.004$.

S: Relative small value of Need Coefficient for the action by Need Coefficient Function with $A = 1.0$, $B = -0.002$.

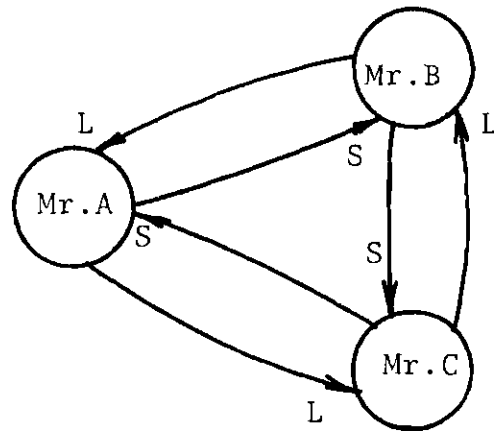


Figure 4-7. Allocation of Different Need Coefficient Functions to Each Participant

(b) Same Need Coefficient (1,1)*. The relationship between Need Coefficient and proficiency is described as follows:

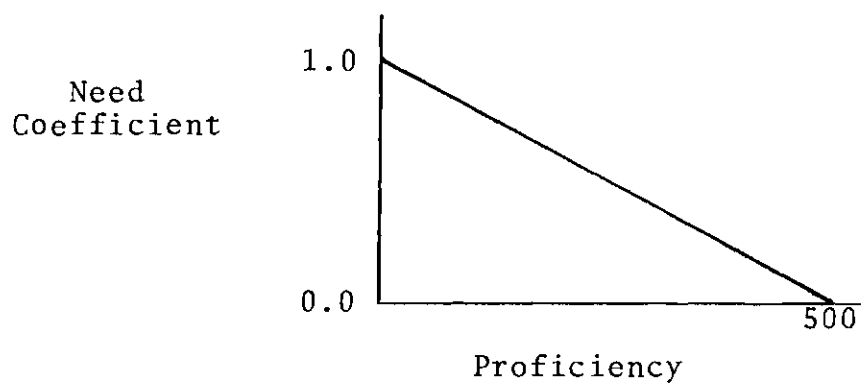


Figure 4-8. Need Coefficient Function Type C

(3) Initial Value of Proficiency (P I/C)*. The simultaneous runs were varied according to the following assumptions.

(a) All initial values of proficiency are equal to zero (0,0)*

SXGBA = 0.0

SXFCA = 0.0

SXEAB = 0.0

SXFCB = 0.0

SXGBC = 0.0

SXEAC = 0.0 (IV-92)

(b) All initial values of proficiency are equal to fifty (50,50)*

SXGBA = 50.0

SXFCA = 50.0

SXEAB = 50.0

SXFCB = 50.0

SXGBC = 50.0

SXEAC = 50.0 (IV-93)

(c) The initial value of proficiency which has a large Need Coefficient Function is one-hundred and the initial value of proficiency which has a small Need Coefficient Function is fifty (100,50)*

SXGBA = 100.0

SXFCA = 50.0

SXEAB = 50.0

SXFCB = 100.0

SXGBC = 50.0

SXEAC = 100.0 (IV-94)

(d) The initial value of proficiency which has a large Need Coefficient Function is fifty and the initial value of proficiency which has a small Need Coefficient Function is one-hundred (50,100)*

SXCBA = 50.0

SXFCA = 100.0

SXEAB = 100.0

SXFCB = 50.0

SXCBC = 100.0

SXEAC = 50.0 (IV-95)

(e) The initial value of proficiency which has a large Need Coefficient Function is three-hundred and the initial value of proficiency which has a small Need Coefficient Function is zero (300,0)*

SXCBA = 300.0

SXFCA = 0.0

SXEAB = 0.0

SXFCB = 300.0

SXCBC = 0.0

SXEAC = 300.0 (IV-96)

(4) Consideration Coefficient (C/C)*. The value of the Consideration Coefficient is given for the calculation of each loop.

- (a) Same Consideration Coefficient (1,1)*
 Value (1.0) is given to two-step loop
 Value (1.0) is given to three-step loop
- (b) Different Consideration Coefficient (1,0.8)*
 Value (1.0) is given to two-step loop
 Value (0.8) is given to three-step loop
- (c) Different Consideration Coefficient (1,0.5)*
 Value (1.0) is given to two-step loop
 Value (0.5) is given to three-step loop
- (d) Considering only two-step loop (1,0.0)*
 Value (1.0) is given to two-step loop
 Value (0.0) is given to three-step loop
- (5) Motivation Function (M/F)*

- (a) Motivation Function Type A*

Motivation Function is almost equal to Situation Function

$$MA.K = SA.K + 1.0 \quad (IV-97)$$

$$MB.K = SB.K + 1.0 \quad (IV-98)$$

$$MC.K = SC.K + 1.0 \quad (IV-99)$$

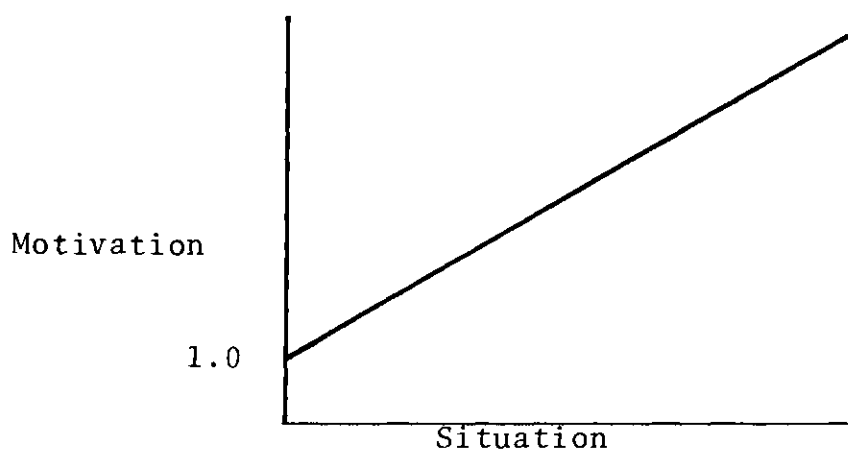


Figure 4-9. Motivation Function Type A

(a) Motivation Function Type B*

Motivation is determined so that thirty-five teaching actions by the participant are derived with maximum action from the system with maximum Need Coefficient (2,1)*, or so that twenty-five teaching actions are derived with maximum action from the system with maximum Need Coefficient (1,1)*.

$$MA.K = 9.8 + (SA.K)(0.23) \quad (IV-100)$$

$$MB.K = 9.8 + (SB.K)(0.23) \quad (IV-101)$$

$$MC.K = 9.8 + (SC.K)(0.23) \quad (IV-102)$$

The following Run List indicates the variations which were incorporated in each simulation run. (See Figure 4-11.)

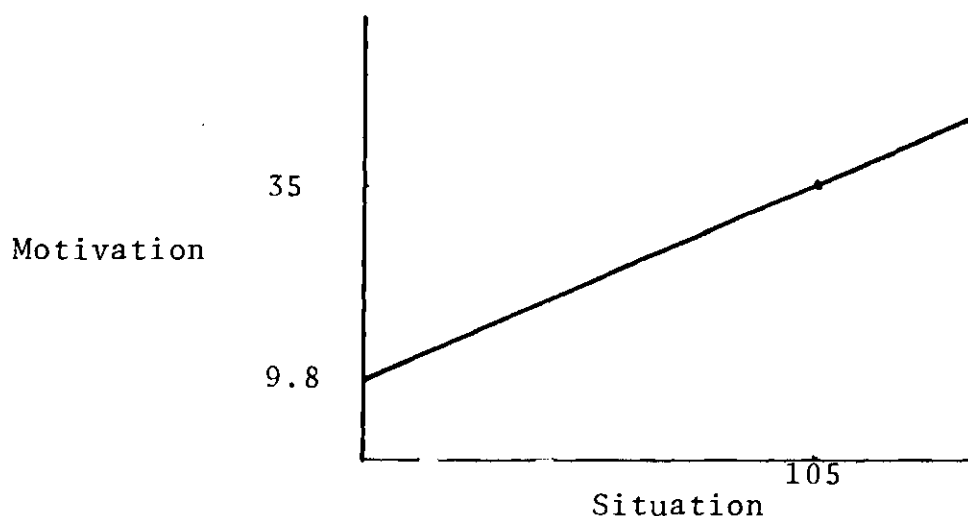


Figure 4-10. Motivation Function Type B

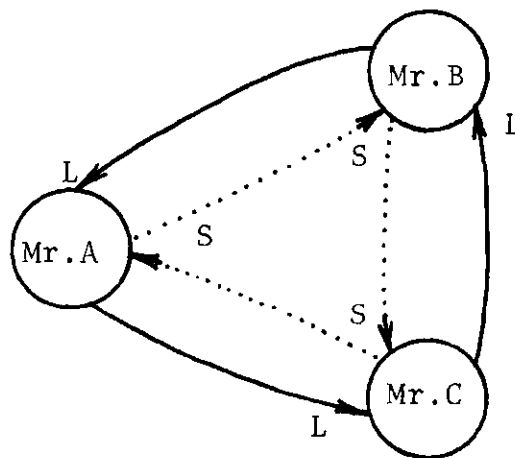
Run	A.D.C.			N/F		P I/C			C/C				M/F		
	N/C	O/V	F/V	2,1	1,1	0,0	50, 50	100, 50	50, 100	1,1	1,0.8	1,0.5	1,0.0	A	B
A-1	x			x		x					x				x
A-2	x			x			x				x				x
A-3	x			x				x			x				x
A-4	x			x					x		x				x
B-1		x		x		x					x				x
B-2		x		x			x				x				x
B-3		x		x				x			x				x
B-4		x		x					x		x				x
C-1			x	x		x					x				x
C-2			x	x			x				x				x
C-3			x	x				x			x				x
C-4			x	x					x		x				x
D-1			x		x	x					x				x
D-2			x		x		x				x				x
D-3			x		x			x			x				x
D-4			x		x				x		x				x
E-1			x	x		x				x					x
E-2			x	x		x					x				x
E-3			x	x		x						x			x
E-4			x	x		x							x		x
F-1		x		x		x				x					x
F-2		x		x		x					x				x
F-3		x		x		x						x			x
F-4		x		x		x							x		x
G-1		x			x	x				x					x
G-2		x			x		x			x					x
G-3		x			x			x		x					x
G-4		x			x				x	x					x
H-1			x		x	x					x				x
H-2			x		x		x				x				x
H-3			x		x			x			x				x
H-4			x		x				x		x				x
I-1	x			x				300,0			x				x
J-1		x		x				300,0			x				x

Figure 4-11. Simulation Run List

Result of Simulation

(1) Comparison of the results by different methods of assuming the Distribution Coefficient for the calculation of gain (Runs A, B, C, E, F) (refer to Figures 4-13, 4-14, 4-15, 4-16, 4-17).

All results show that the actions XEAC, XGBA, XFCB are more valuable than the actions XEAB, XFCA, XGBC, (refer to Figures 4-13, 4-14, 4-15, 4-16, 4-17), that is, by acting indirectly through a third person, each person is receiving the action which he needs the most, as illustrated in Figure 4-12.



————: Strong action

-----: Weak action

L: Large value of Need Coefficient

S: Small value of Need Coefficient

Figure 4-12. Indirect Action Through a Third Person

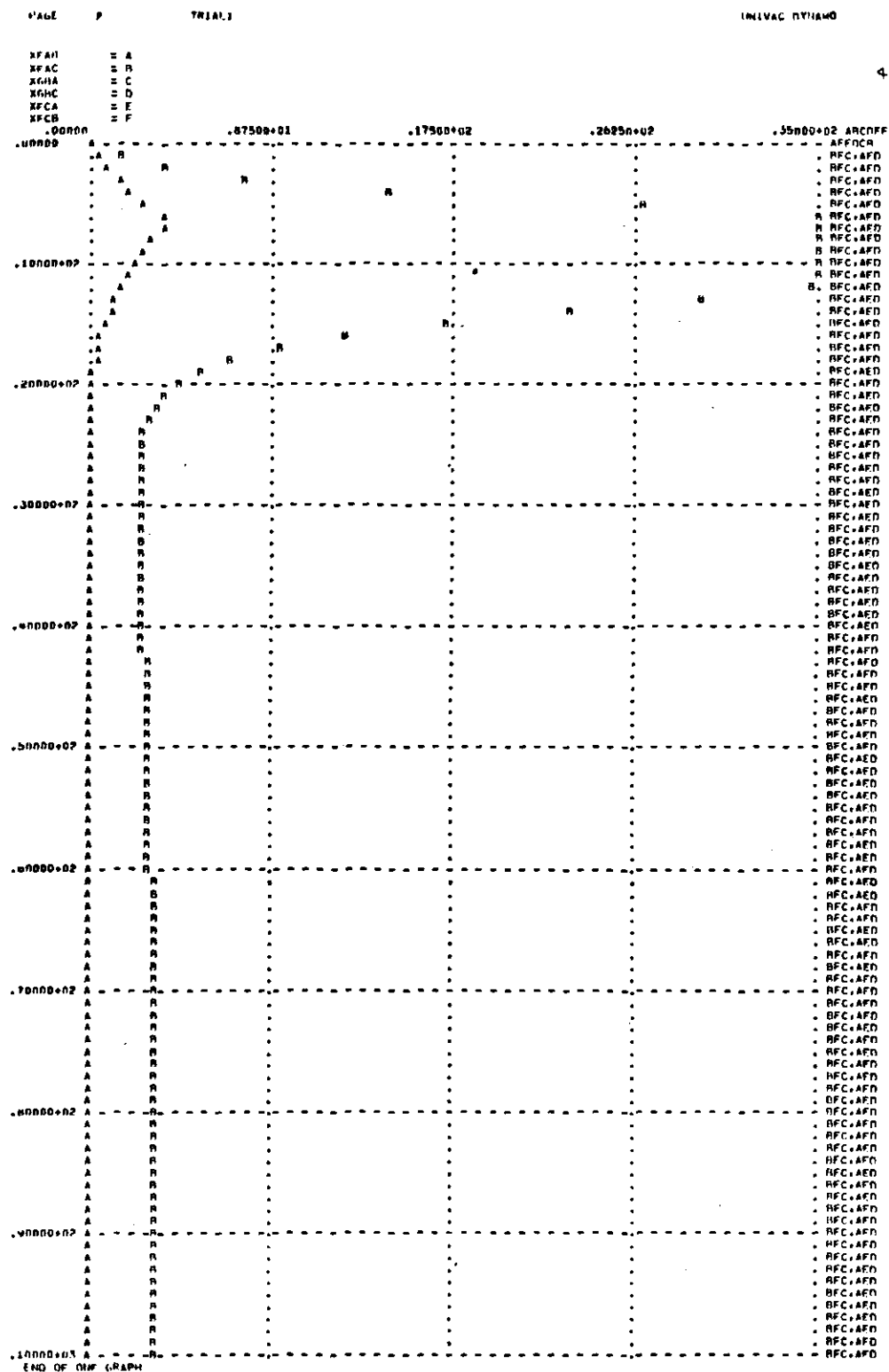


Figure 4-14. Run B-1

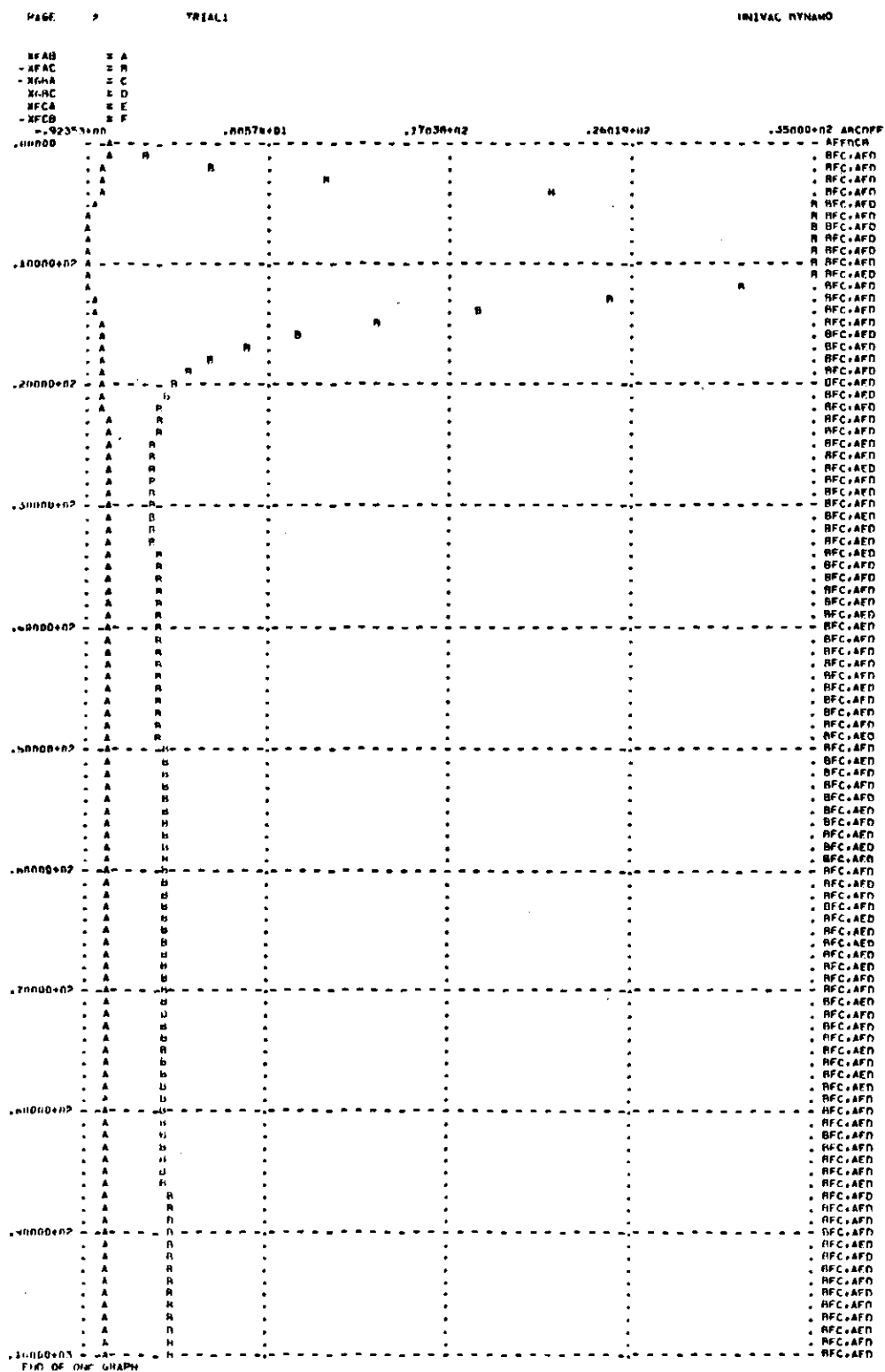


Figure 4-15. Run C-1

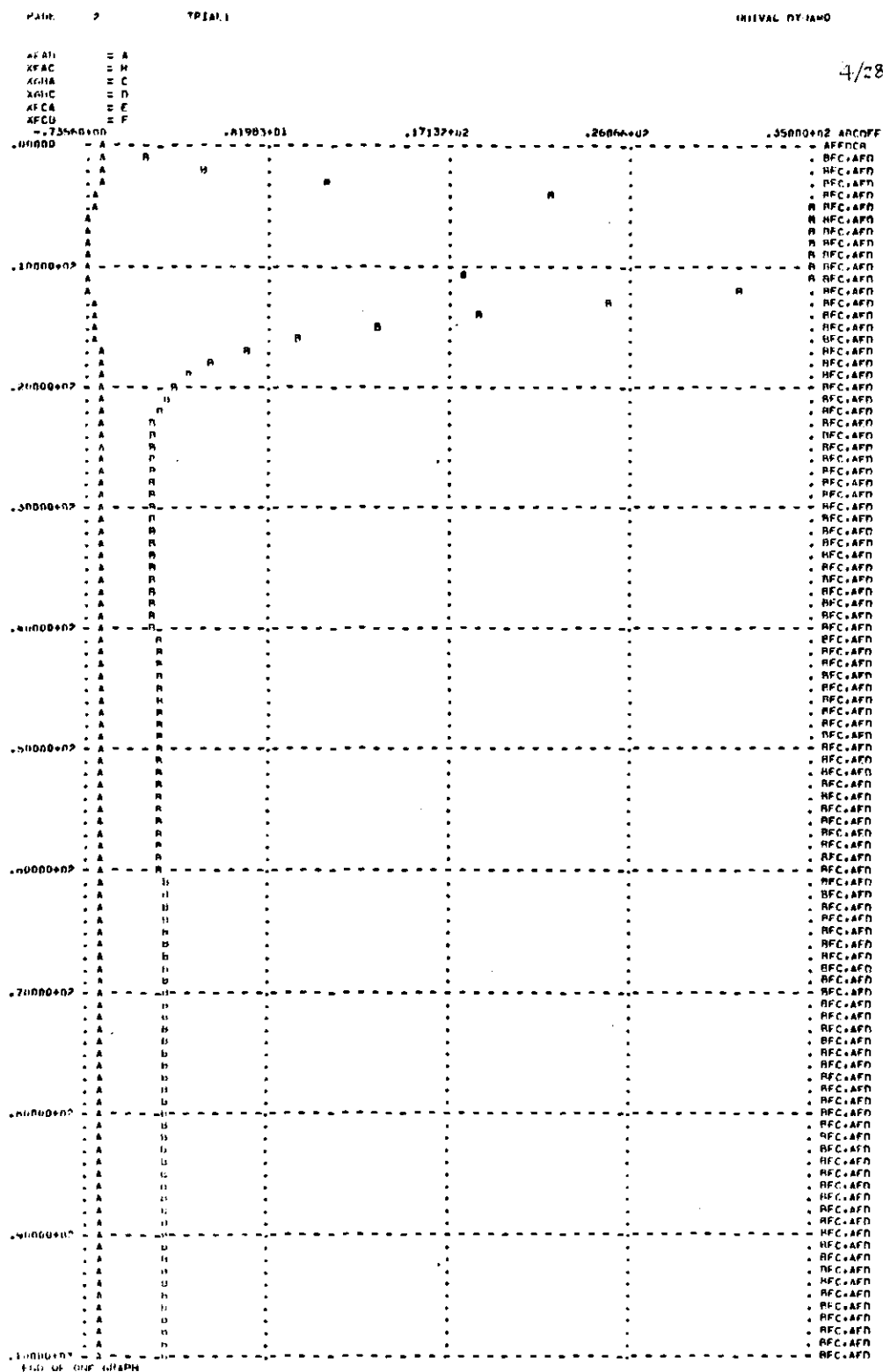


Figure 4-16. Run E-1

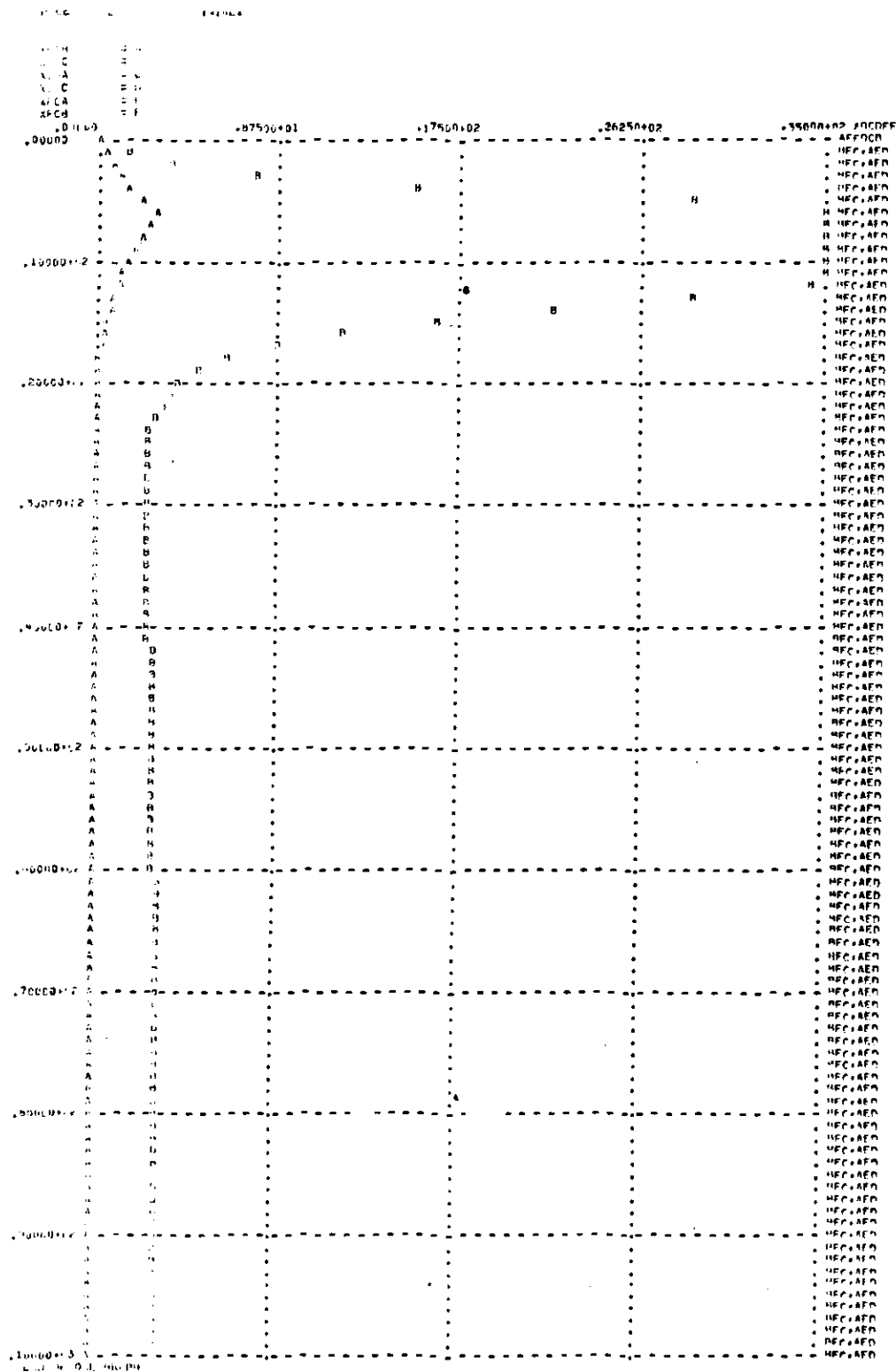


Figure 4-17. Run F-1

But the dynamic behavior of the action varies depending on the method of assuming the Distribution Coefficient (refer to Figures 4-14, 4-15, 4-16, 4-17).

In the case of the method based on Need Coefficient (Run A), at the twentieth week in Run A-1, all actions are taking the same value. At this point, each action XEAB, XGBC, XFCA is increasing by lagging XEAC, XGBA, XFGB (see Figure 4-13). Because the increase of proficiency (see Figure 4-18) due to each action XEAC, XGBA, XFGB is decreasing its Need Coefficient, then the Need Coefficient of each action XEAB, XGBC, XFCA is increasing relatively; therefore, the expected Distribution Coefficient for each action XEAB, XGBC, XFCA is increasing (refer to Figure 4-19).

At the seventh or eighth week, (in Figure 4-13), actions XEAB, XGBC, XFCA are at their maximum values and the growth rate of proficiency is also at its maximum value (in Figure 4-18). After the twentieth week in Figure 4-18, the actions have a small value because of the small value of the Situation Function (in Figure 4-20); then proficiency gradually decreases by forgetting and the overshoot of behavior is modified to arrive at the equilibrium value.

On the other hand, in the case of the method based on an observed value of the Distribution Coefficient (Runs B, F), the observed value is determining the Distribution Coefficient and the tendency of the Distribution Coefficient is escalated to an extreme point, $K = 1.0$ or $K = 0.0$. This

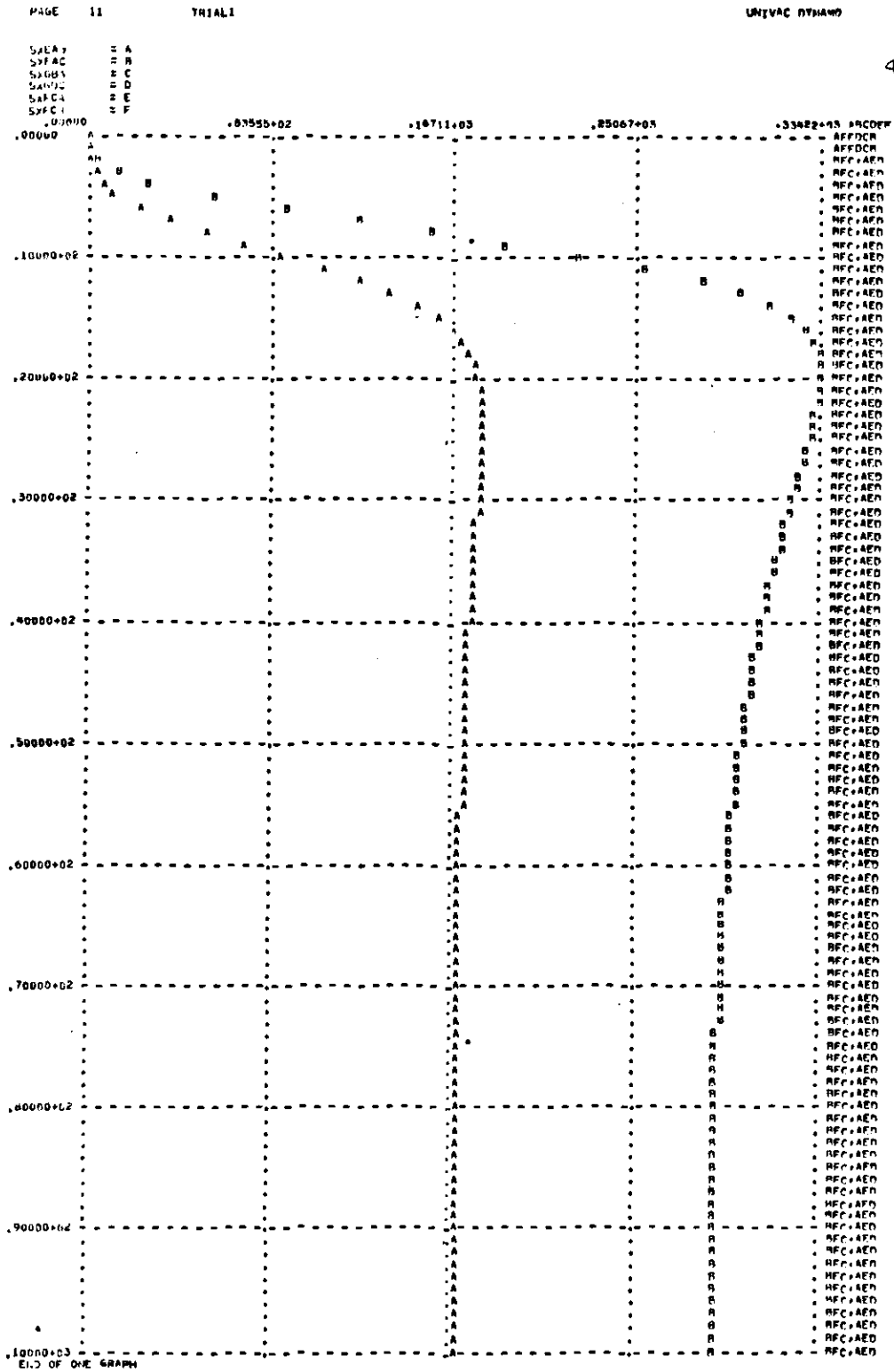


Figure 4-18. Run A-1

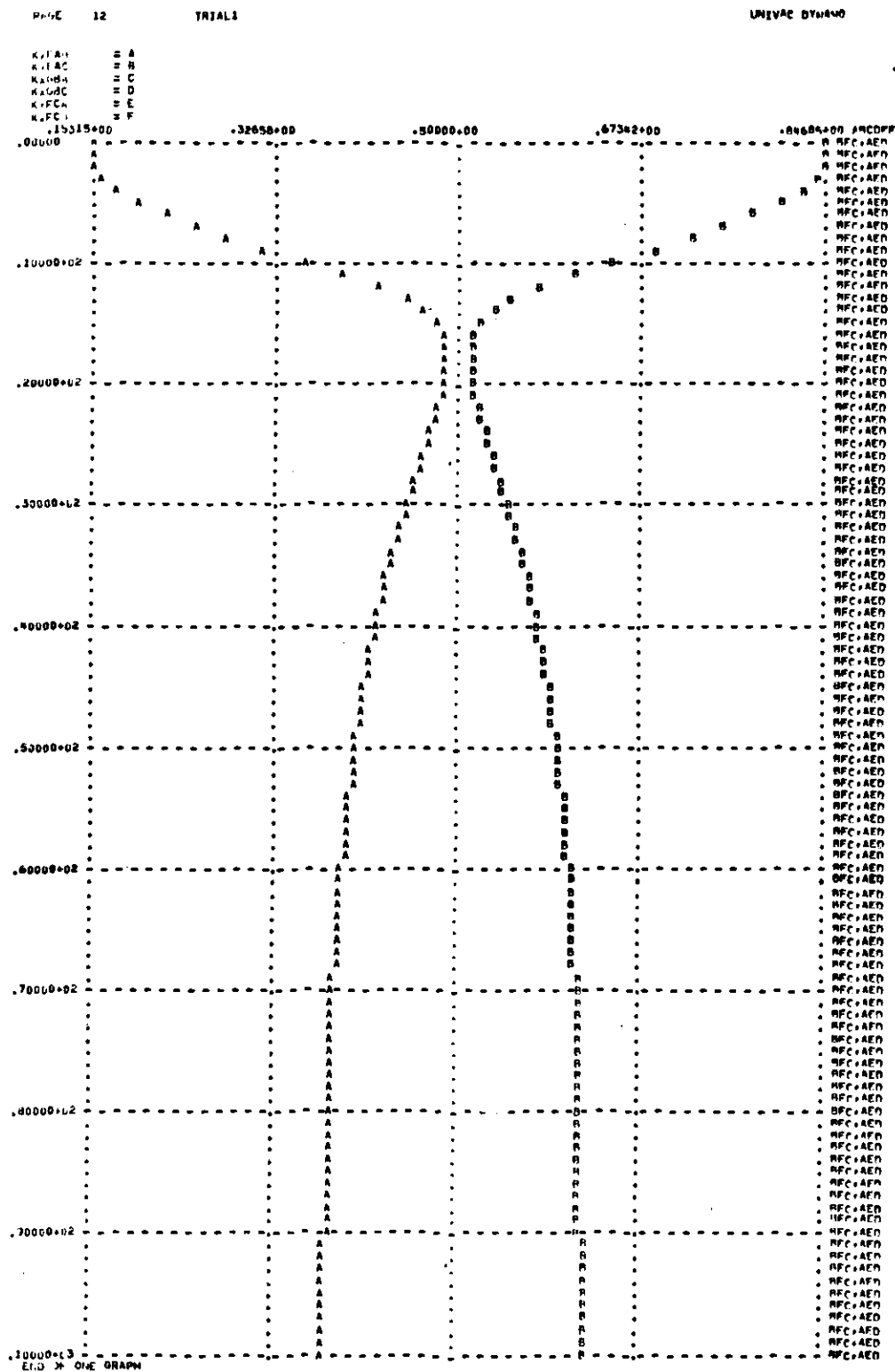


Figure 4-19. Run A-1

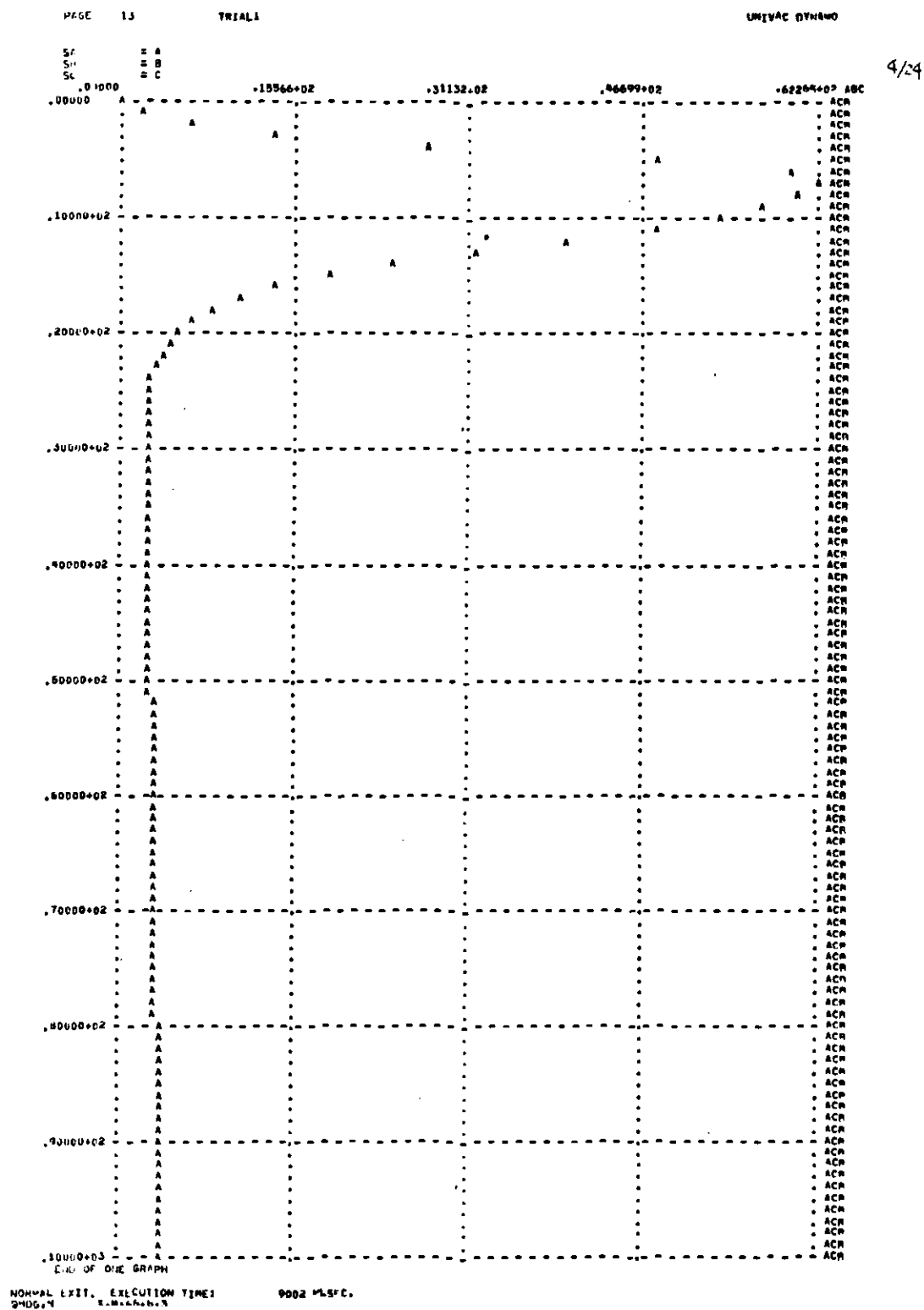


Figure 4-20. Run A-1

This characteristic is observed in a Distribution Coefficient curve (refer to Figures 4-21, 4-22). Each Distribution Coefficient KXEAC, KXGBA, KXFCA is increasing in value to $K = 1.0$ and KXEAB, KXFCA, KXGBC are decreasing in value to $K = 0.0$. Momentary increases of XEAB, XGBC, XFCA, are due to the rapid growth of the Situation Function.

In the case of the method based on a tendency of the Distribution Coefficient (Runs C, E), a positive loop for the change in the Distribution Coefficient is constructed. After the Distribution Coefficient reaches an extreme point, (see Figures 4-23, 4-24) the behavior of each action depends strongly on the value of the Situation Function (refer to Figures 4-15, 4-28).

(2) Comparison of the results by different Need Coefficient Functions (Runs C,D). When the Need Coefficient is defined by the same function, the value of the Distribution Coefficients is a constant value $K = 0.5$. Then, the behavior of each participant is perfectly symmetrical. (Refer to Figures 4-25, 4-26, 4-27).

The value of the Situation Function with a large value of the Need Coefficient is naturally larger than the value of the Situation Function with a small value of the Need Coefficient; also, each action is different (see Figures 4-28, 4-29) corresponding to the gain value for each action because of a different Need Coefficient.

(3) Comparison of the results by different initial

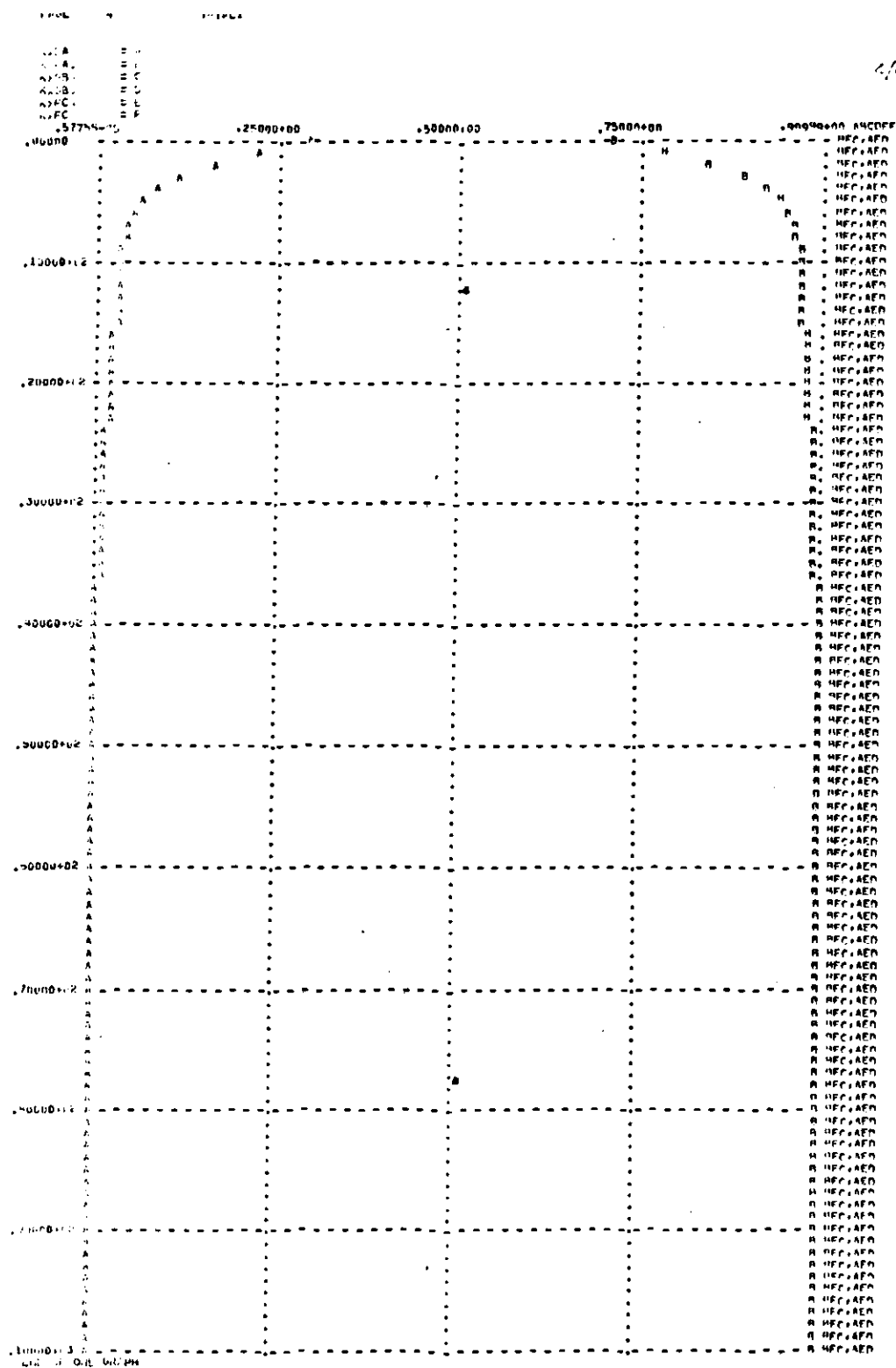


Figure 4-22. Run F-1

27a

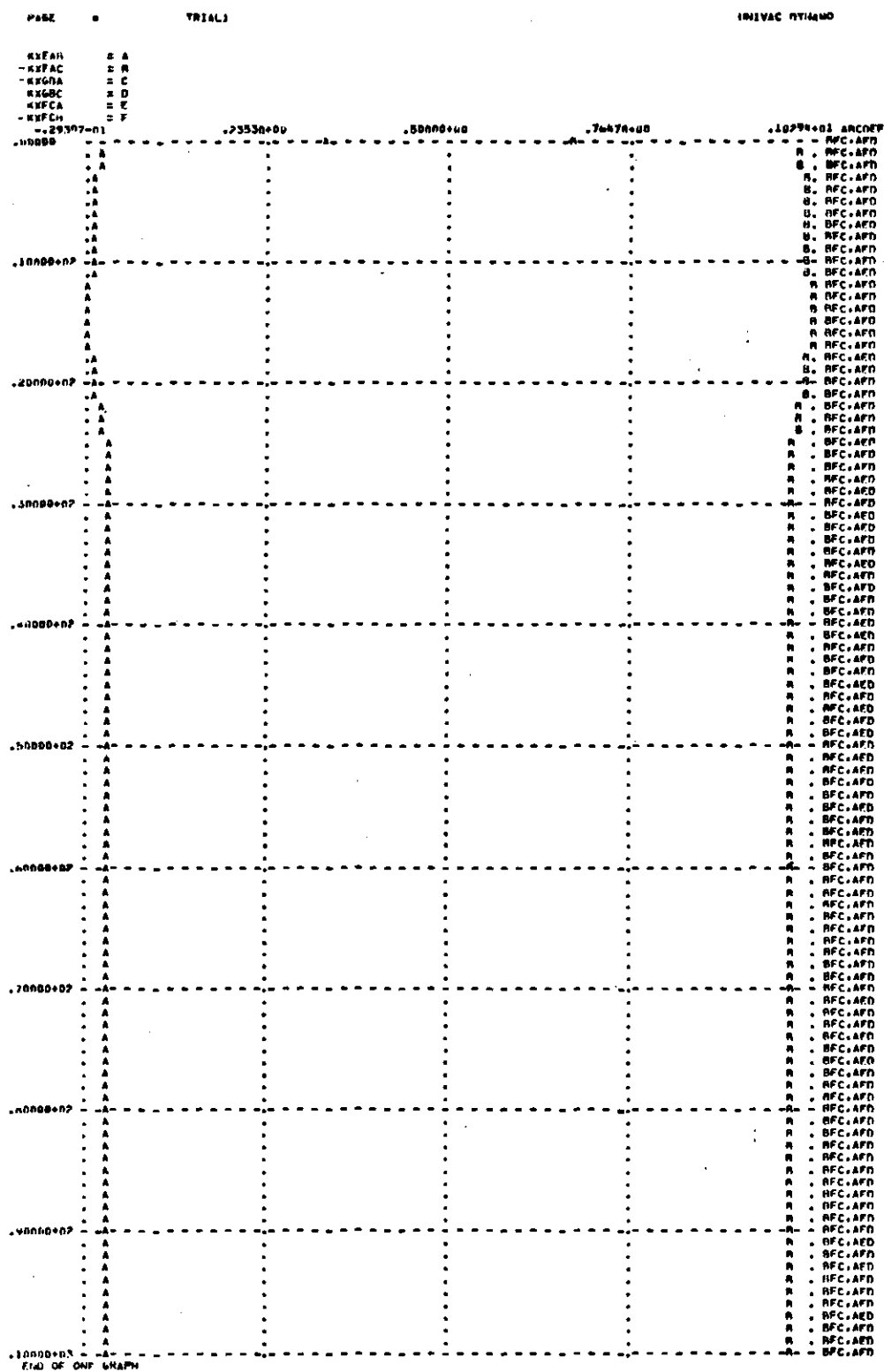


Figure 4-23. Run C-1

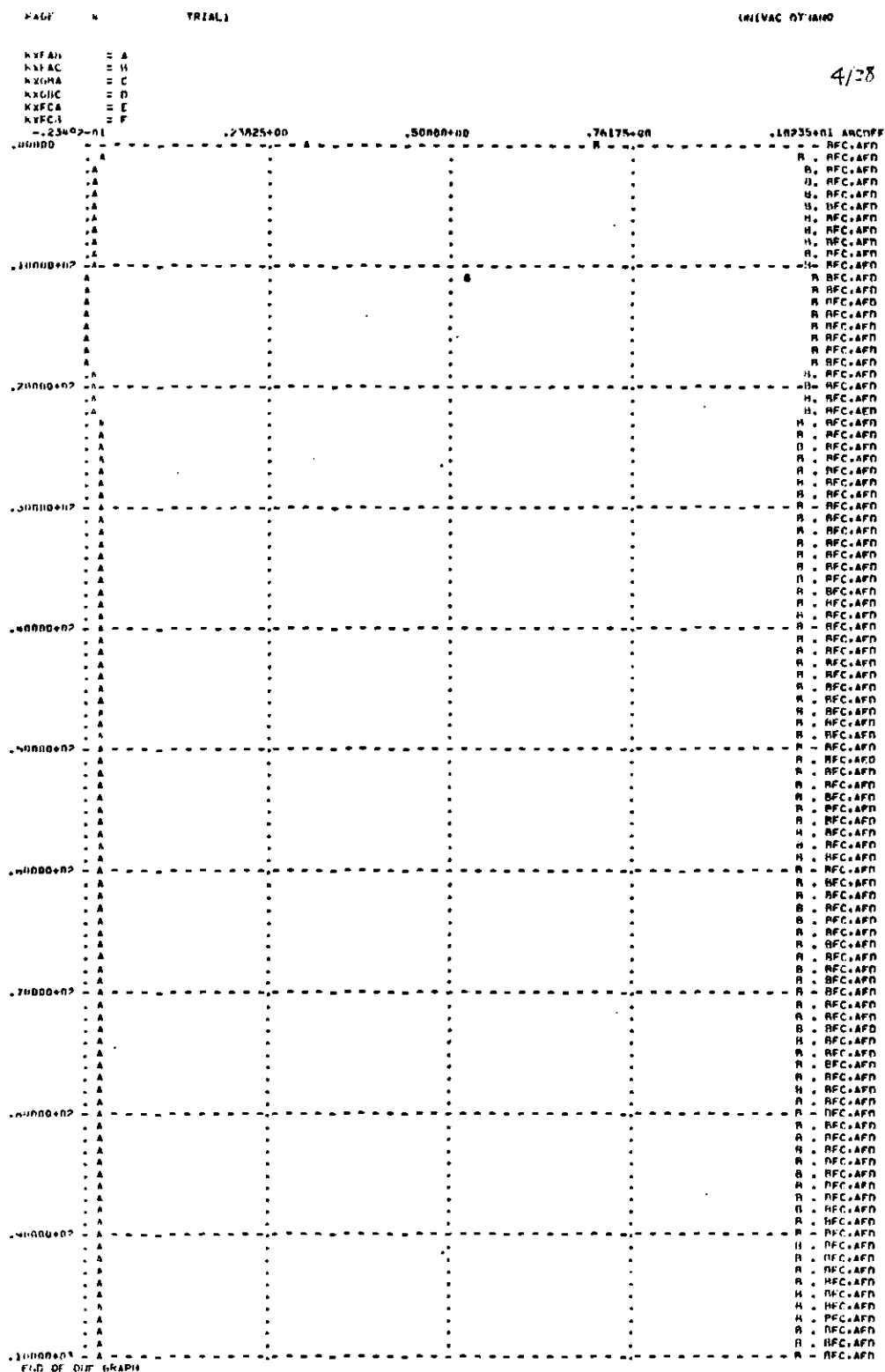
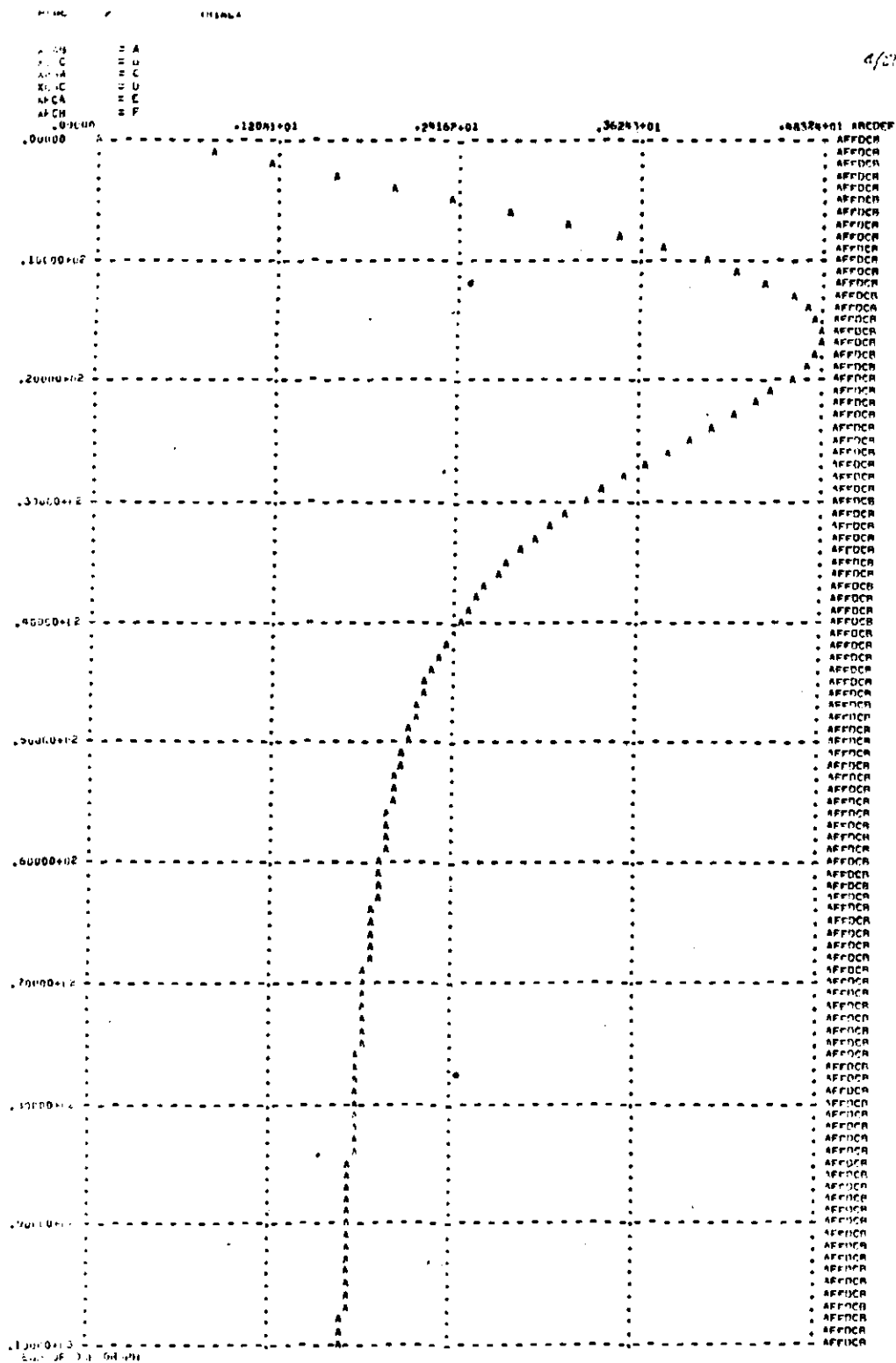


Figure 4-24. Run E-1



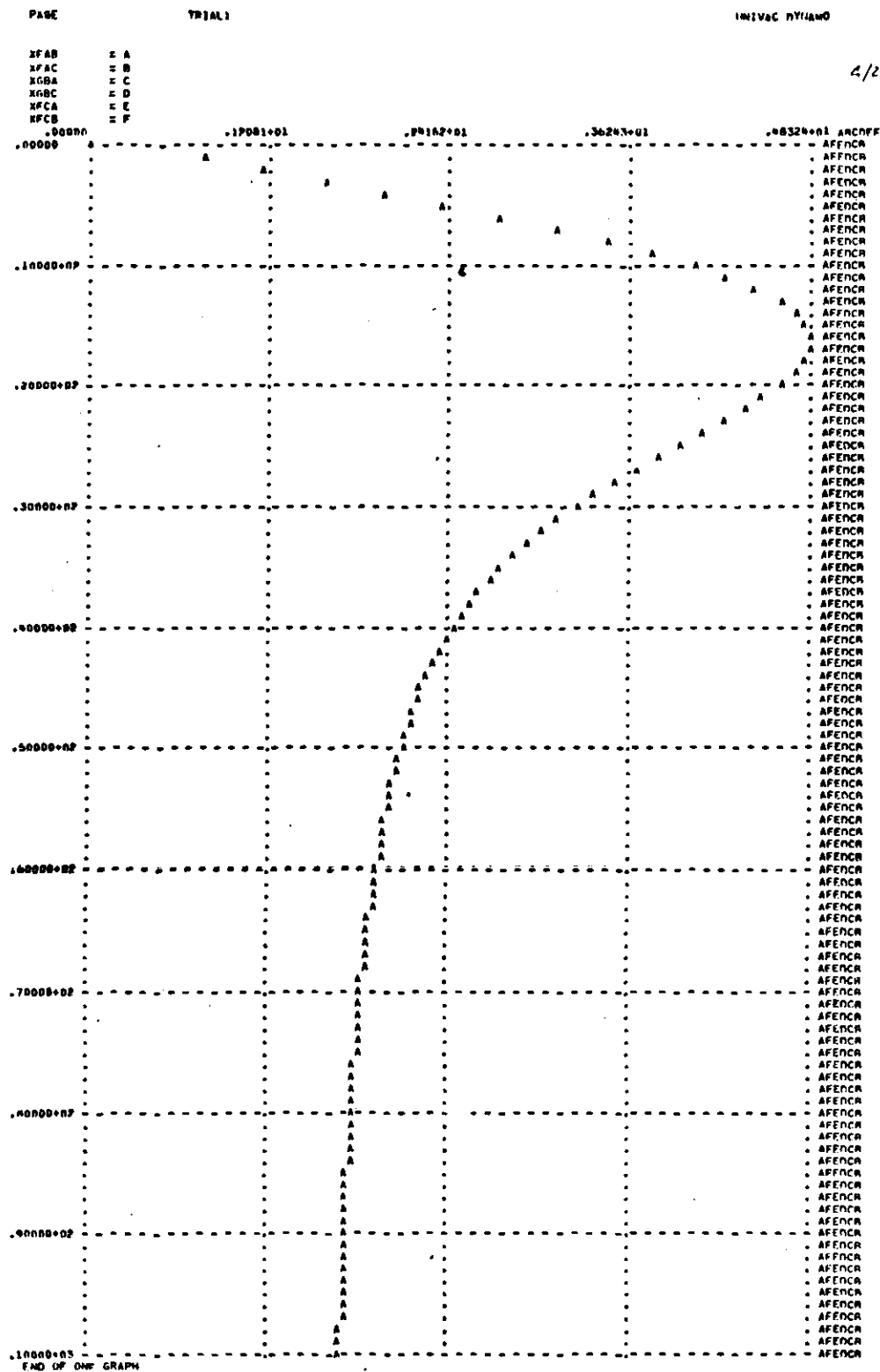


Figure 4-26. Run G-1

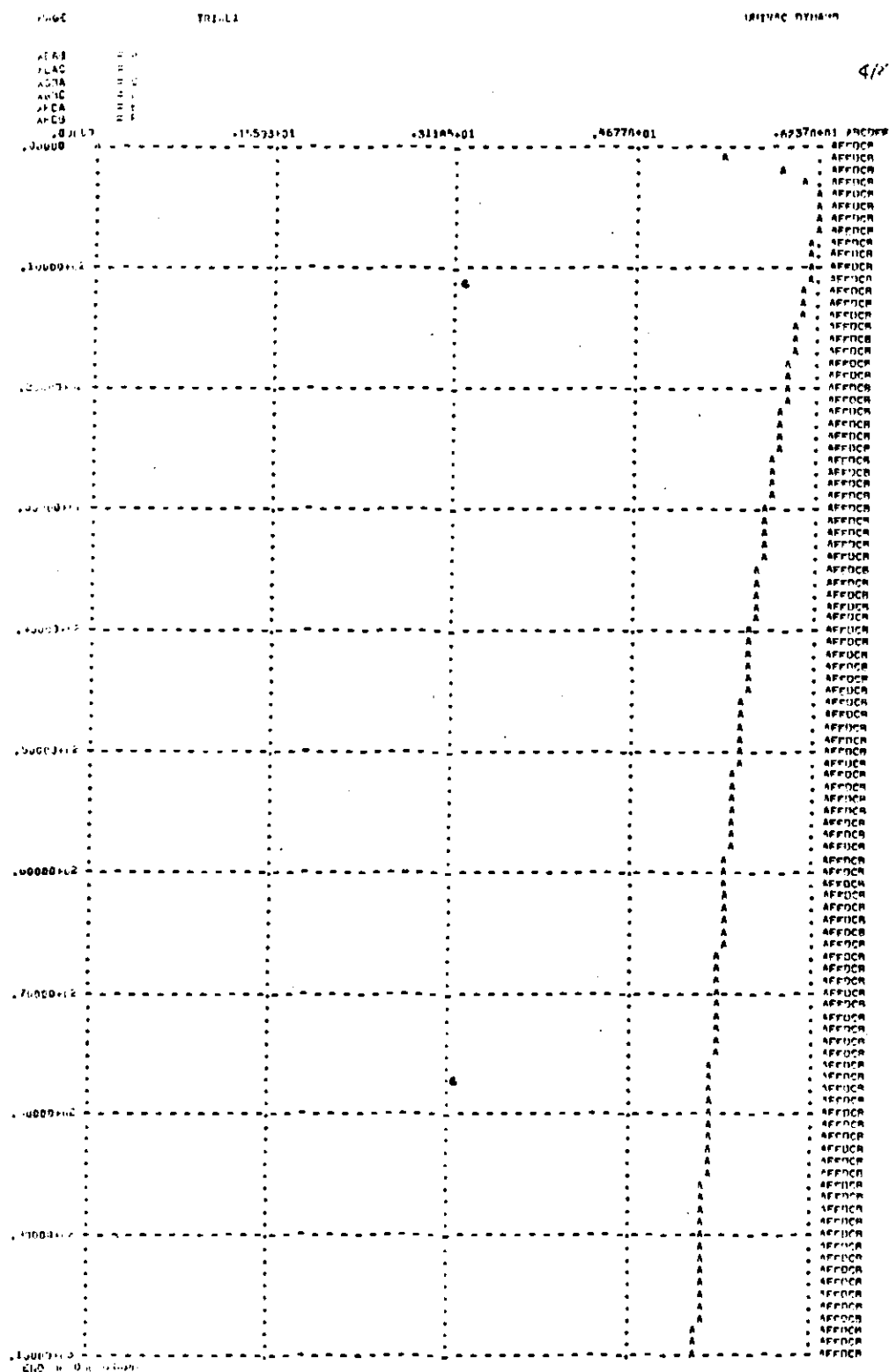


Figure 4-27. Run H-1

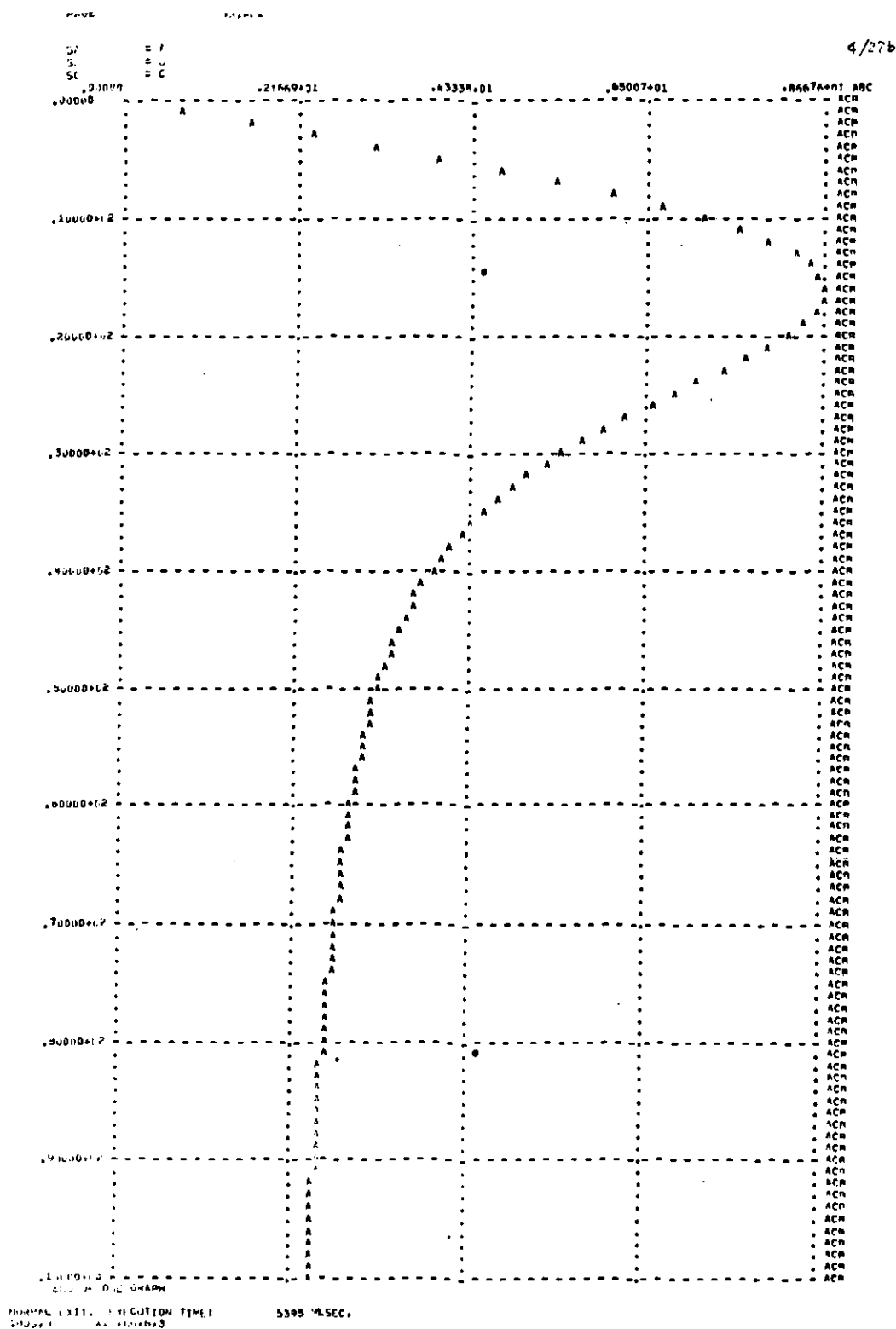


Figure 4-29. Run D-1

values of proficiency. No difference in the equilibrium value of proficiency is observed. Only transient behavior is different. (Refer to Figures 4-13, 4-31, 4-32, 4-33.) In Run A the Need Coefficient for a large initial value of proficiency is small, then, in order to satisfy the need for the action with a large value of the Need Coefficient $XEAB$, $XGBC$, $XFCB$ are larger than $XFAC$, $XGBA$, $XFCA$ at first. (Refer to Figures 4-30, 4-34).

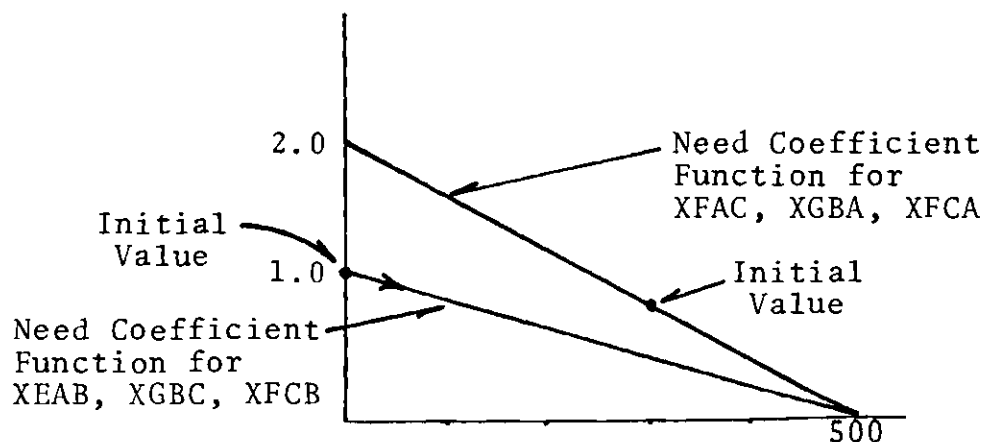


Figure 4-30. Relationship Between Two Different Need Coefficient Functions

After arriving at the same Need Coefficient by receiving the actions with a large value of Need Coefficient (at the thirty-third week in the Run), a clear shift of action is observed in the action curve and Distribution Coefficient Curve (refer to Figure 4-35). This kind of behavior appears with delay in Run J (see Figures 4-36, 4-37)

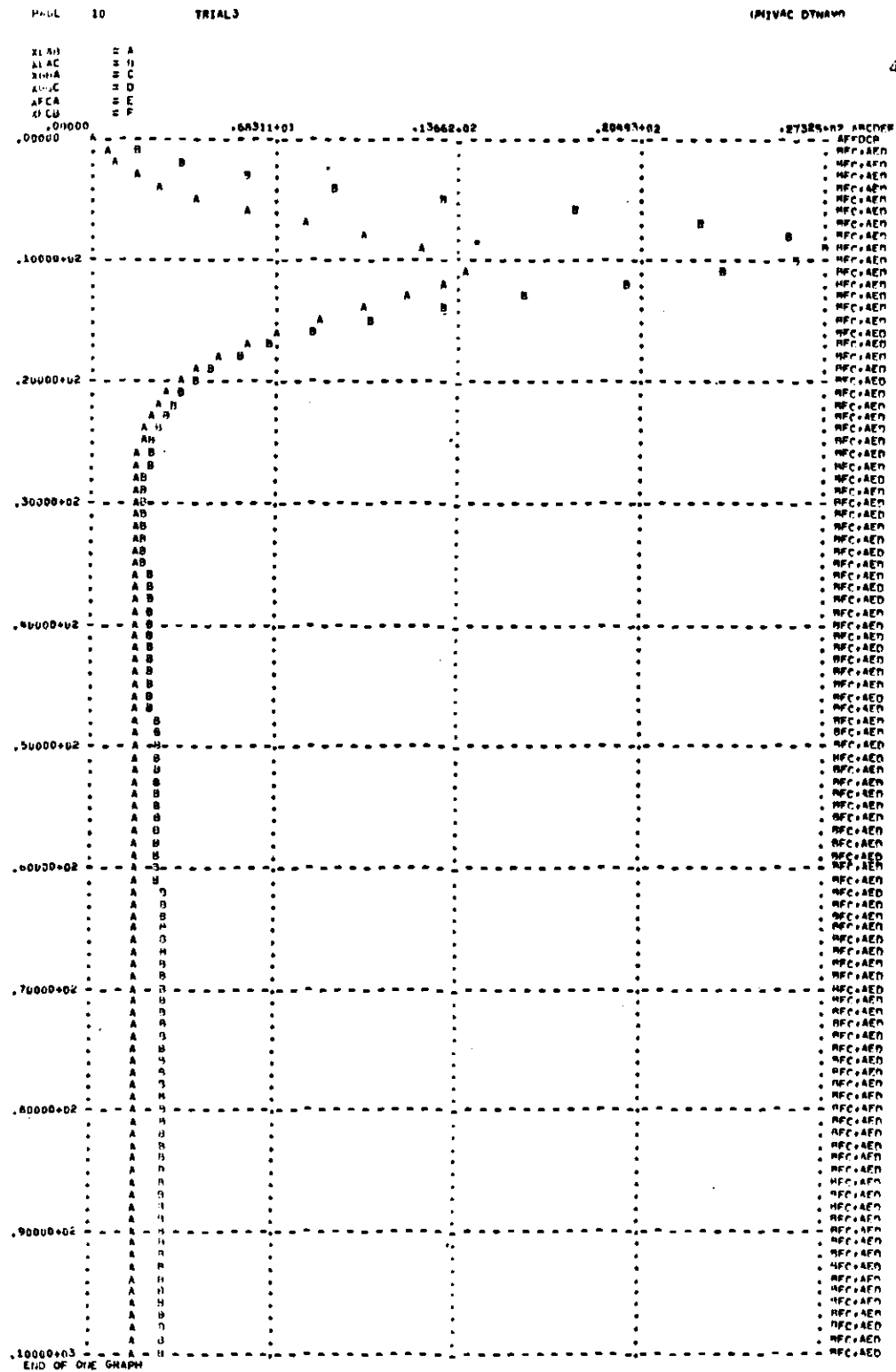


Figure 4-32. Run A-3

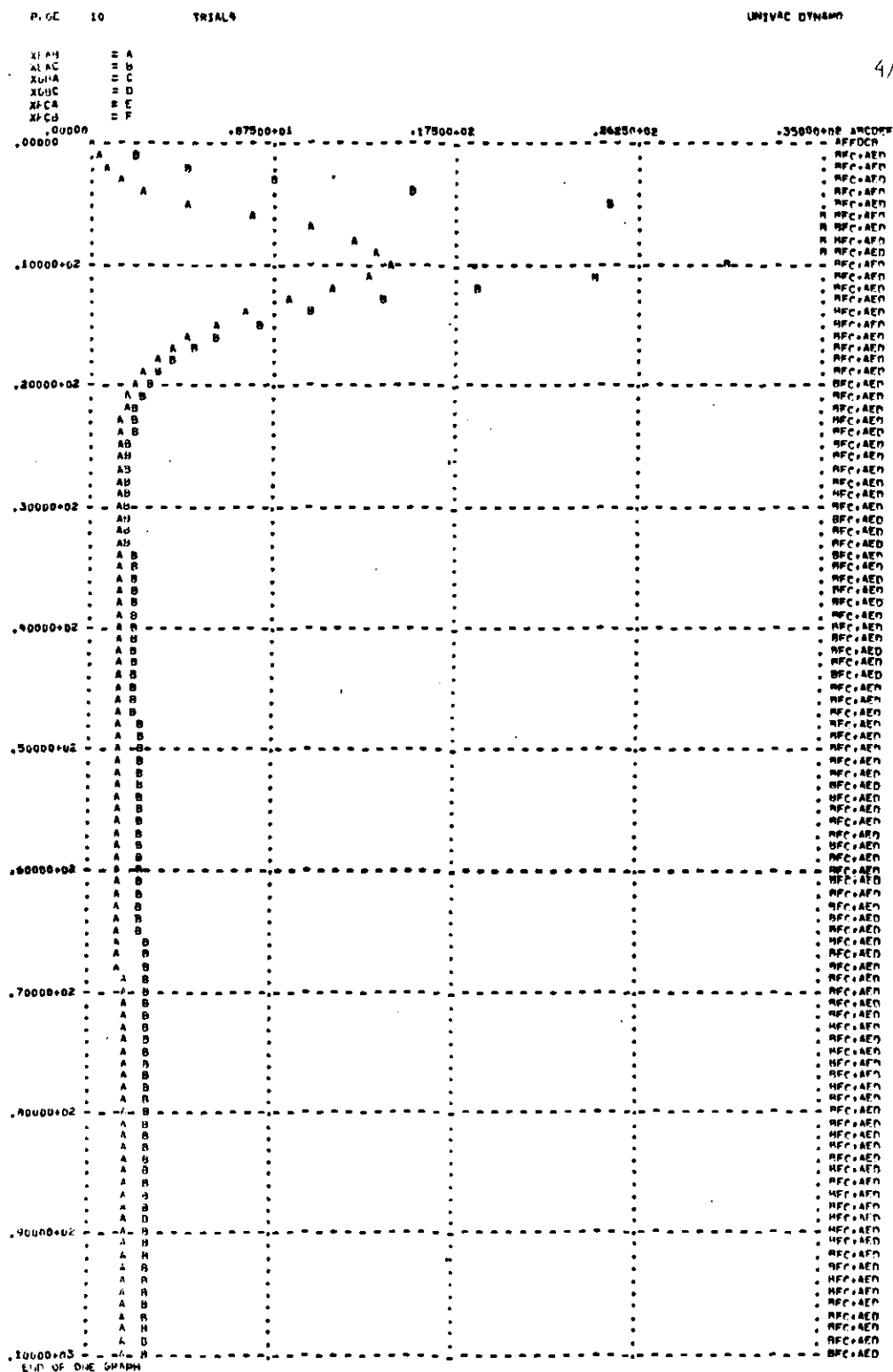


Figure 4-33. Run A-4

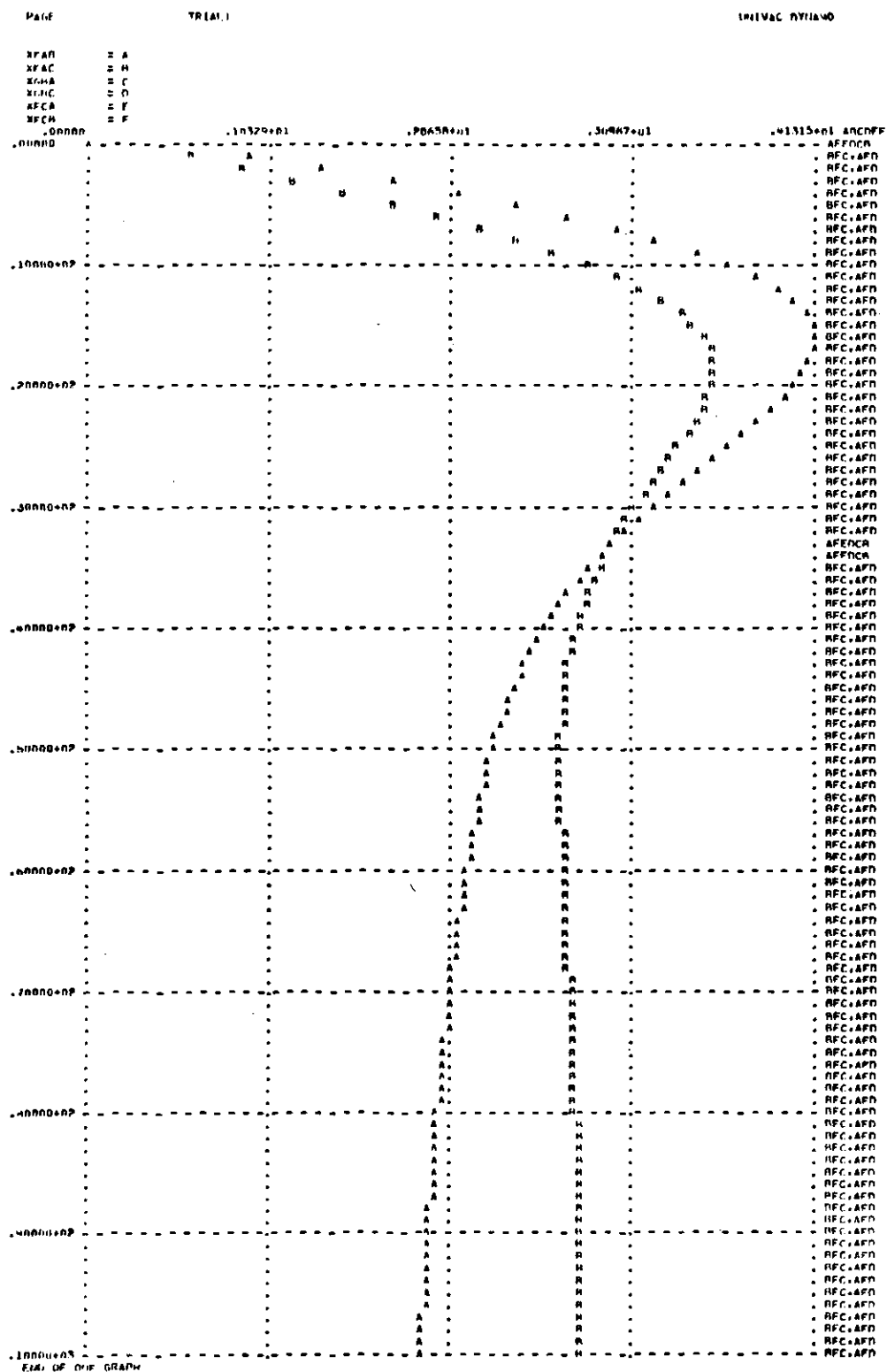


Figure 4-34. Run I-1

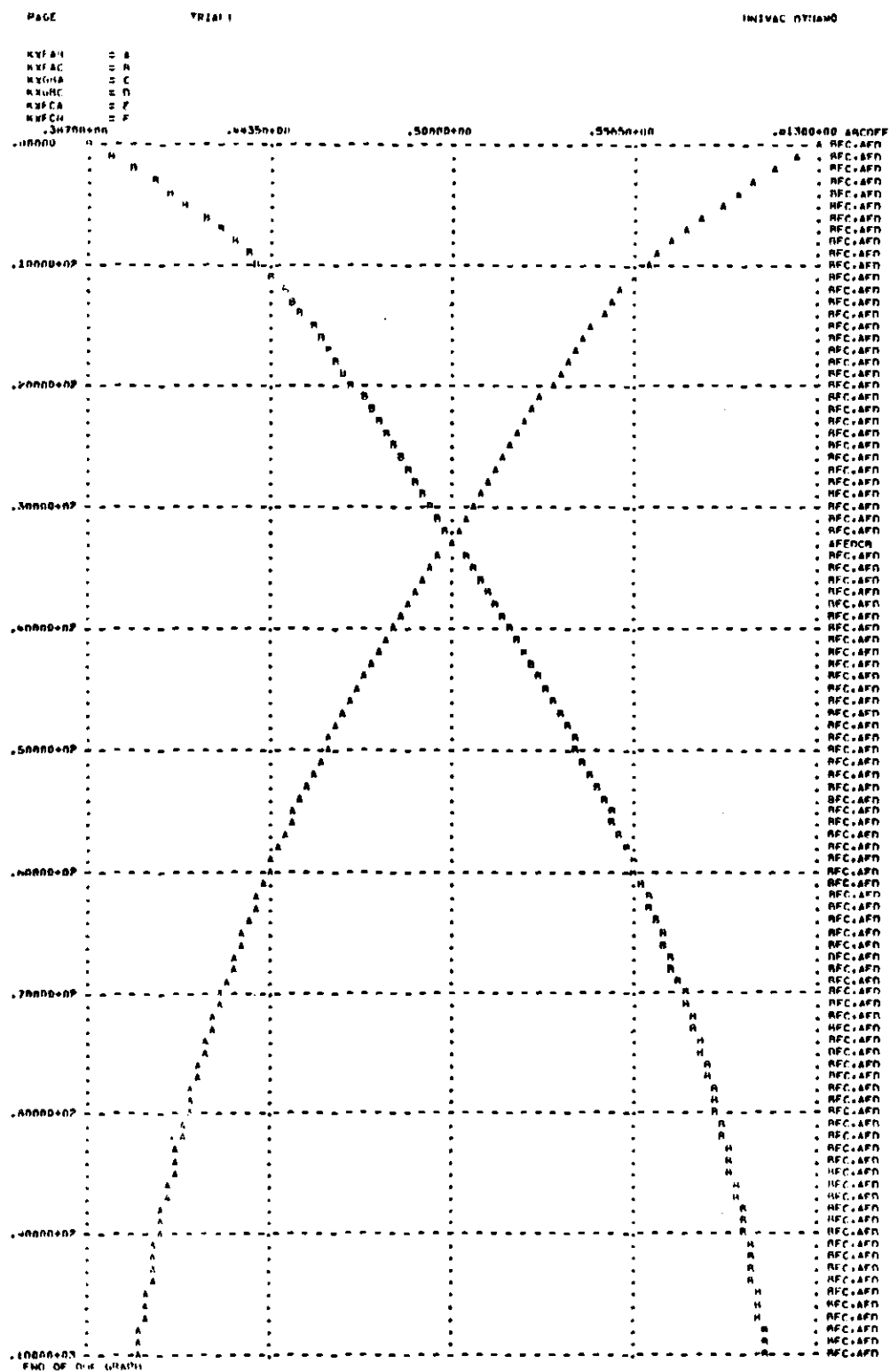


Figure 4-35. Run I-1

because the past value of the Distribution Coefficient affects calculation of gain; then, even if the value of the Need Coefficient for the actions XEAB, XGBC, XFCA is sufficiently smaller than the value of the other actions XEAC, XGBA, XFGB, each action XEAB, XGBC, XFCA is still larger than XEAC, XGBC, XFGB, respectively (from the fifteenth week to the fifty-sixth week) because the old large Distribution Coefficients for XEAB, XGBC, XFGB still affect the gain calculations.

(4) Comparison of the results by different consideration Coefficients (Runs E,F). A clear difference is observed when the decision maker ignores the three-step loops. (Refer to Figures 4-38, 4-41.) In this simple example, when the decision maker considers only the two-step loop (see Figures 4-39, 4-40), there is no difference in gain value; then, the result of evaluation of the two actions of each participant is the same. Therefore, all actions have the same value. Since in this example, the gain is very large where the three step loop is considered, then obviously it is not appropriate to ignore the three-step loop for decision making. This is the good example which explains the effect of consideration on the result (refer to Figures 4-38, 4-39, 4-40, 4-41).

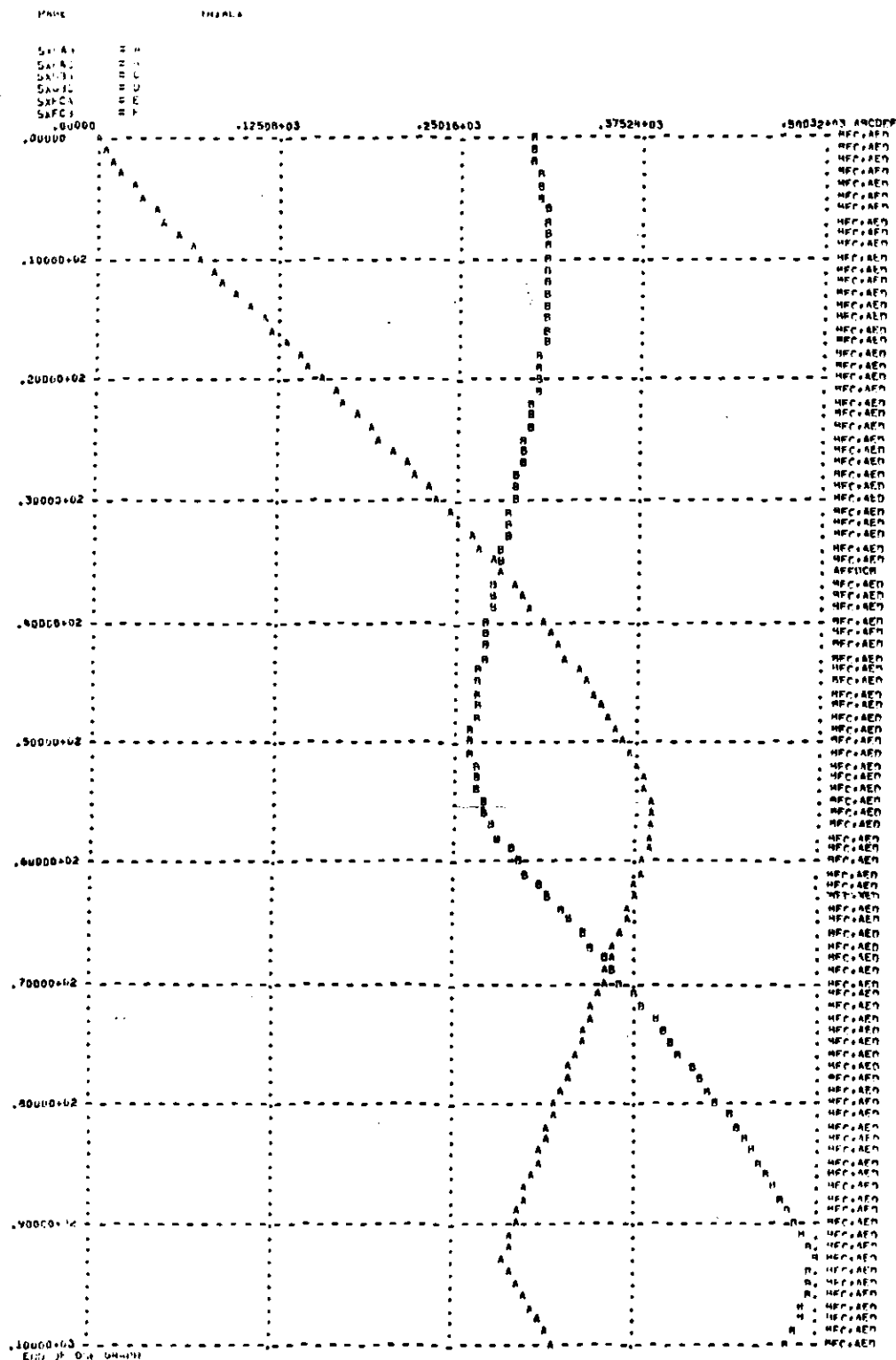


Figure 4-37. Run J-1

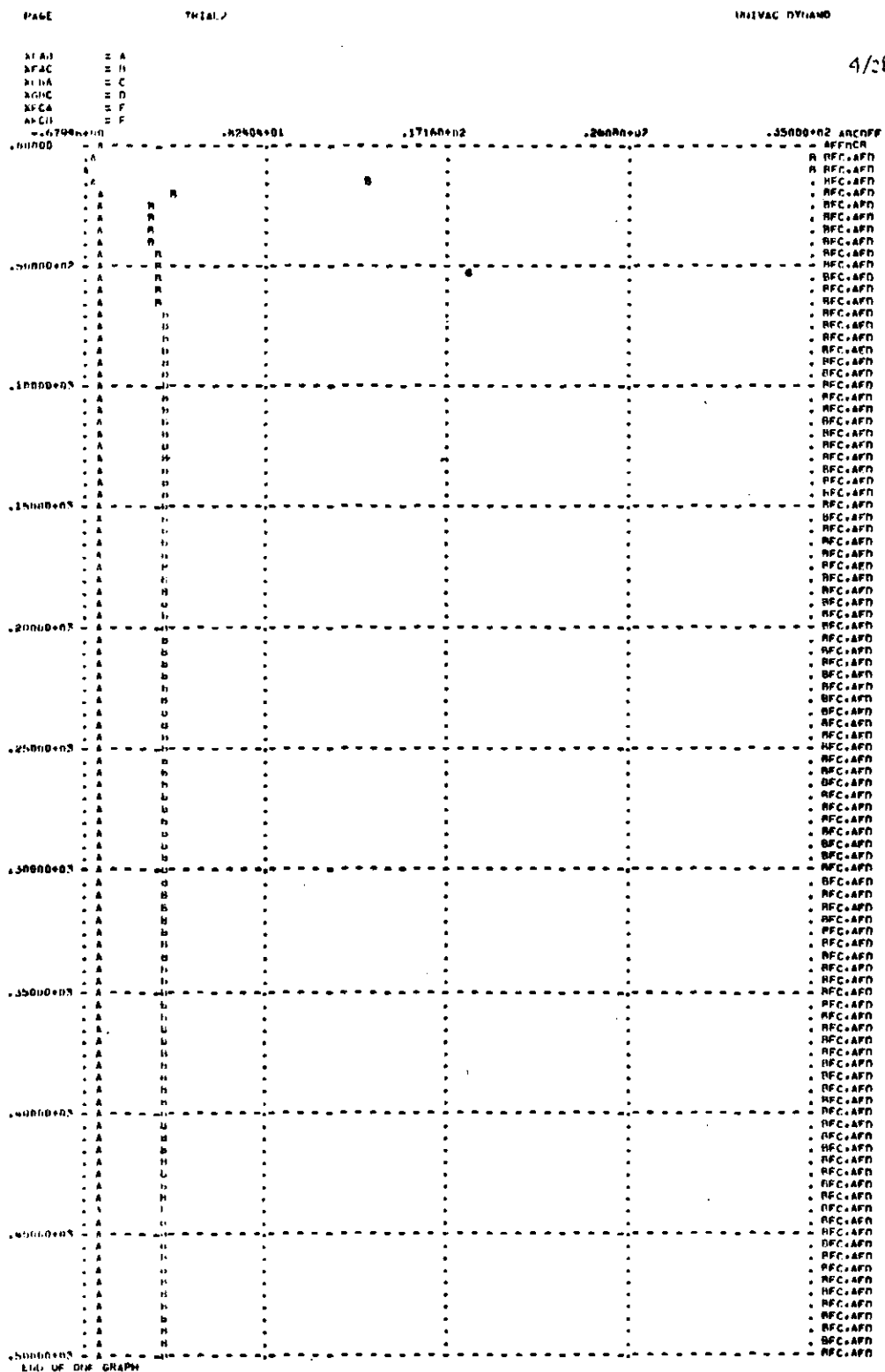


Figure 4-38. Run E-1

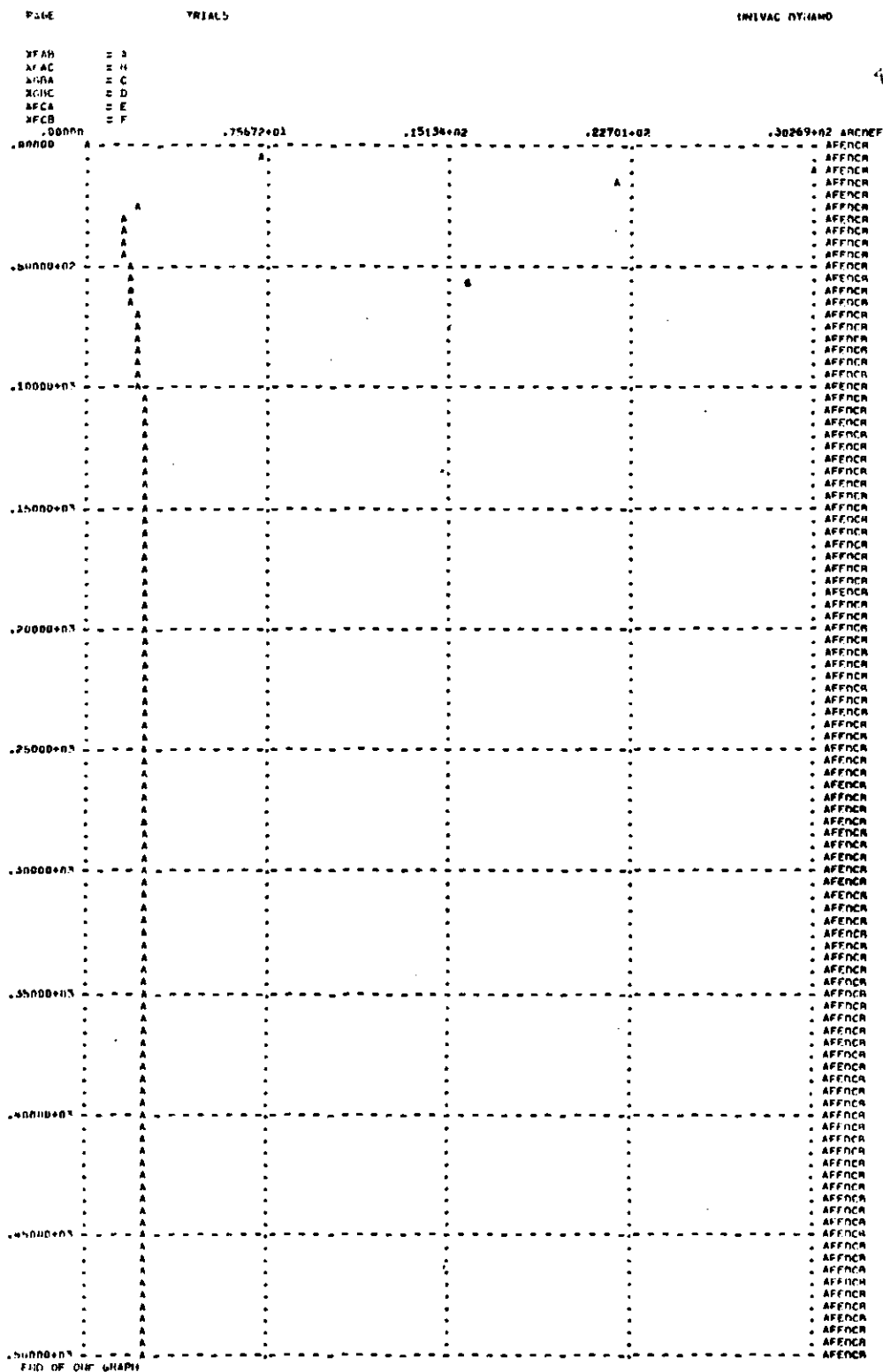


Figure 4-39. Run E-4

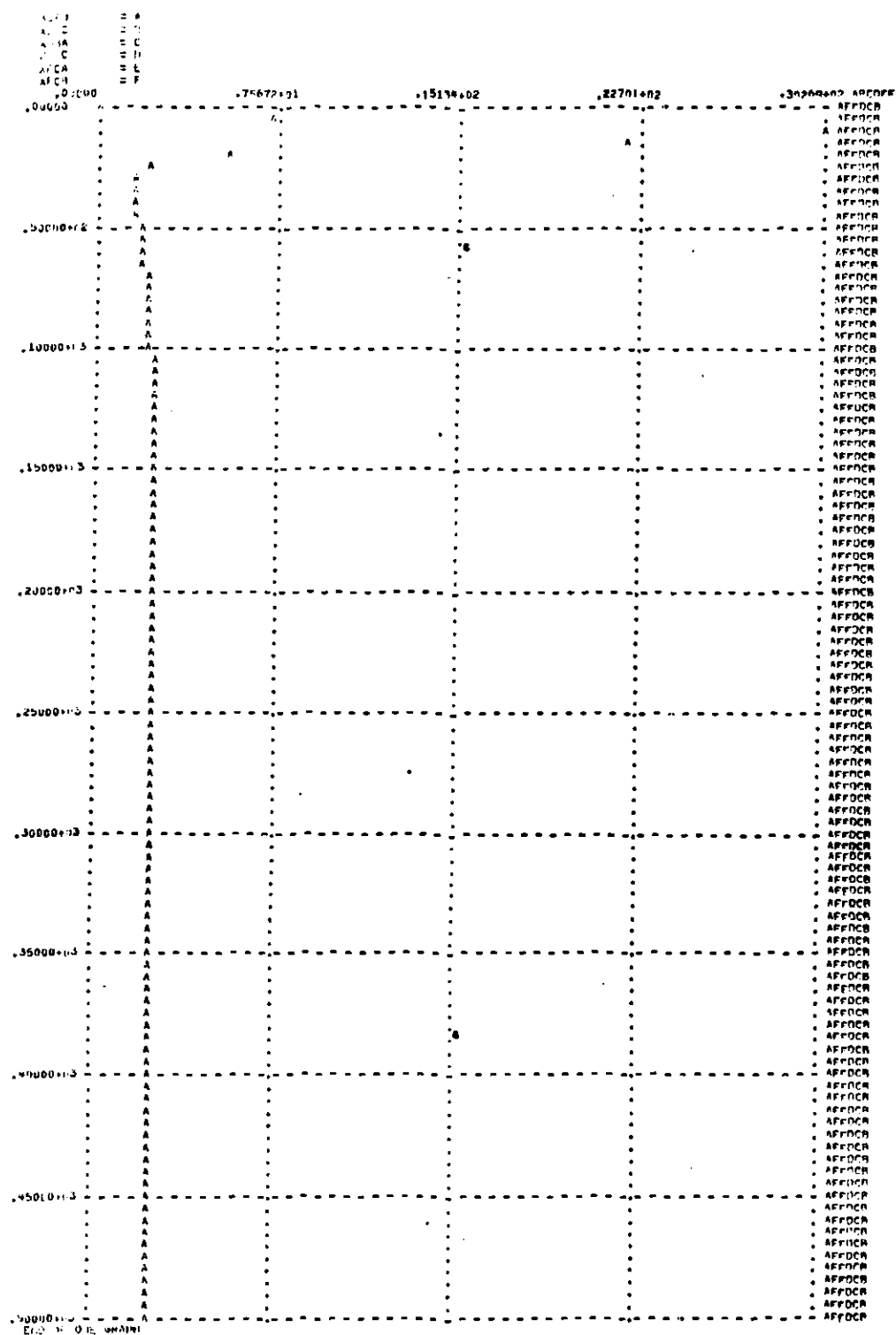


Figure 4- 40. Run F-4

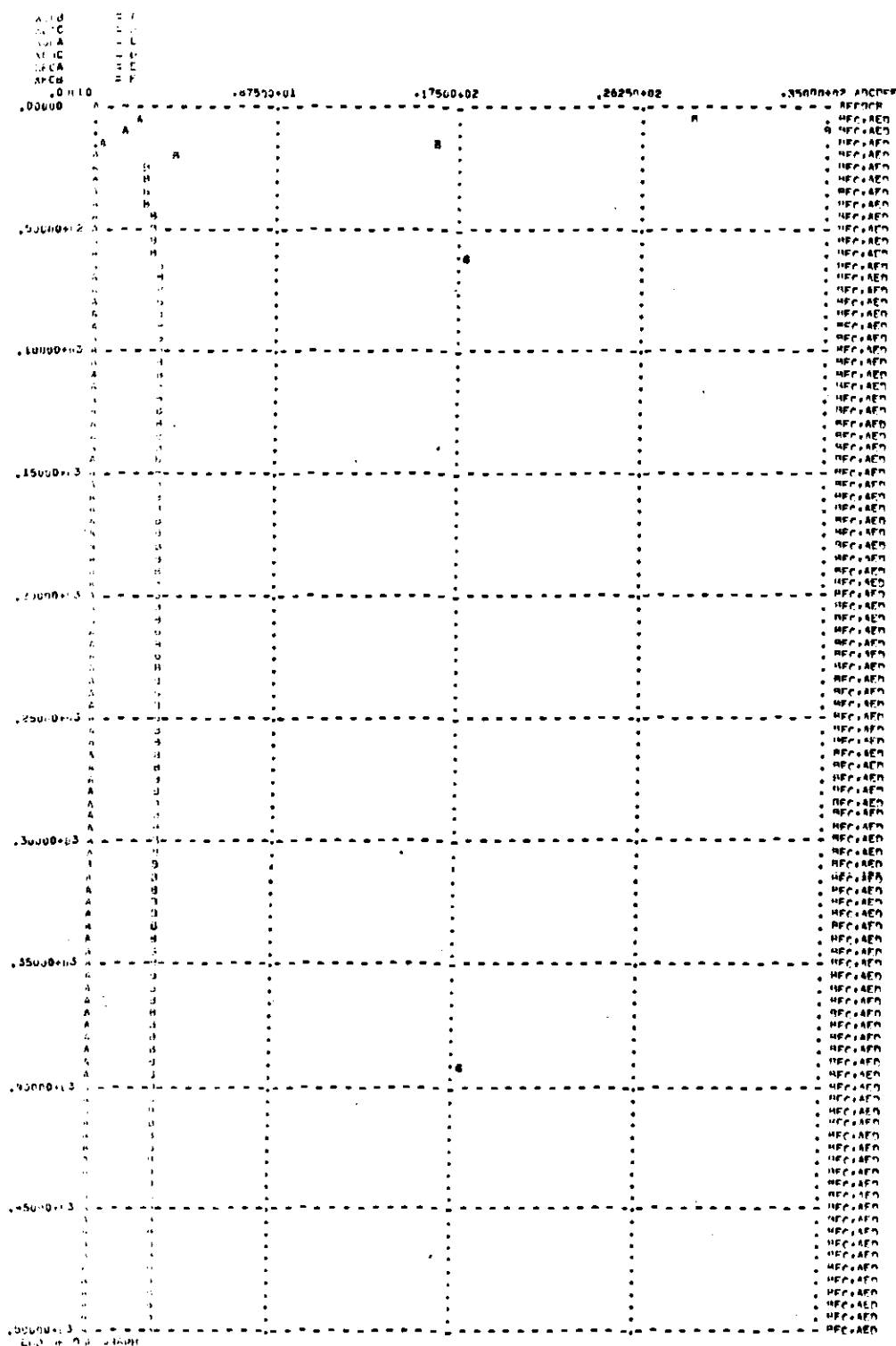


Figure 4-41. Run F-1

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Results of the Research

This research has produced an analytic model of decision making within the framework of dynamic multi-product exchange systems. Some significant features of this model are the following:

- (1) Each participant tries to maximize his own Situation Function, or level of satisfaction.
- (2) Before a participant acts, he predicts the reaction from the others to each of his possible actions.
- (3) The needs of each participant are affected through time by the actions he receives.
- (4) Three different ways of predicting the reactions of participants are available.
- (5) Varying levels of complexity of the reactions are handled explicitly by the use of Consideration Coefficients.

The simulation of this general exchange system model, using the different decision making procedures available, led to widely differing results in system behavior. Similar differences occurred among the procedures under a variety of conditions. Thus, it is possible to draw some general

conclusions regarding the influence of decision making upon system behavior.

1. The consideration of only two-way reaction loops in the decision making process excluded certain forms of three way exchange among the participants. If the more desirable actions among three participants could be obtained only by such an exchange--A to B, B to C, and C to A--then the simple decision making process led to inferior system behavior. Consideration of the three-way loops generally led to the improved system behavior, with each participant enjoying higher values of his Situation Function.

2. Predicting the reaction of other participants by forecasting linearly based on recent trends resulted in cases where actions were continued long after they ceased contributing to Situation Functions. The forecasted Distribution Coefficients of the participants' actions were, in effect, perpetuating the old system behavior. In some cases the old, inappropriate system behavior continued to the point of system collapse.

3. By considering the potential mutual benefits from nonexistent or low-valued exchanges, significantly improved system behavior can be achieved. This implies that negotiation among several participants can lead to better solutions than assuming that each participant will continue to react as he has in the past.

These conclusions agree generally with intuitive

knowledge about behavior of participants in an exchange system. The advantage of the analytic model is that it makes possible more detailed study of such systems. Also, a number of nonintuitive results concerning systems can be explained in simple terms using the model. For example, the persistence of actions that no longer provide satisfaction to the participants is due to giving too much weight to observed or forecasted Distribution Coefficients of reaction.

Recommendations for Future Work

By applying the model developed in this research to the simple exchange, the advantages and the shortcomings of this approach have been made clear. Accordingly, the directions in which this approach should be developed are outlined below, as follow.

1. Motivation Function

The relationship between the Situation Function and the Motivation Function should be determined empirically.

In the plausible model used in this research, it was assumed that the Motivation Function was originally related to the Situation Function as follows:

$$\text{Motivation} = \text{Sum of the absolute values of the product terms in the Situation Function} + \text{Constant} \quad (\text{V-1})$$

$$M_{pa} = \sum |\text{Need Coefficient} \times \text{Observed Action}| + C \quad (\text{V-2})$$

In fact, many kinds of relationships are possible between the Motivation Function and the Situation Function. In general, the Motivation Function can be defined as:

$$M_{pa} = f(S_{pa}) \quad (V-3)$$

The particular relationship must be determined by a careful sociological and psychological analysis of each participant's behavior in a real problem. Empirical work on such relationships will contribute much to the study of exchange systems.

2. Perceived Model

In an exchange system, each participant can be considered to form a "Perceived Model" of the system for decision making purposes. The utilization of this perceived model depends on the participant's characteristics and ability. Estimating the system reaction by calculating the gain using the Perceived Model is one example where the Perceived Model plays a significant role. A better knowledge of how participants create and use such a Perceived Model is needed.

3. Distribution Coefficient

In this research the Distribution Coefficients for action are determined in proportion to the value of gain for each action. In general, the Distribution Coefficients should be determined as a function of the estimated gain

$$\text{Distribution Coefficient} = f(\text{the result of estimating system reaction in the Perceived Model}) \quad (\text{V-4})$$

Many types of functions can be applied here. For example, it is possible to assume that the rate of change of action is in proportion to the value of gain, and this assumption will be more appropriate for the nature of some participants.

4. Assumed Distribution Coefficient

Three typical ways to determine the Assumed Distribution Coefficients were presented in this research. In the simple model presented in Chapter IV, different behavior patterns corresponding to these different ways were observed. In each case studied the method of determining Assumed Distribution Coefficients was the same for each participant. Different combinations are possible in a real system, however, and these should be studied.

5. Feedback Loop in a System

As explained in Chapter III, there are many types of feedback loops which may define the Need Coefficients. These feedback loops play a significant role in the dynamic behavior of each participant in a system. The model studied had the Need Coefficient for each action related to the accumulated value of that particular action. There may be feedback loops from the accumulated values of other actions,

however, An example of this is where action from A to B arouses jealousy in C, and a higher Need Coefficient for some action from A to C. Future research in this area would help explain the behavior of such systems.

6. General Systems Theory Approach

By attributing to each participant similar functions it may be possible to apply the mathematics of general systems theory to dynamic, multi-product exchange systems. The model so constructed is likely to be exceedingly large. Nevertheless, some useful results may be derived concerning stability and controllability.

APPENDICES

APPENDIX A
LIST OF VARIABLES

XEAB	English teaching action by Mr. A to Mr. B
XEAC	English teaching action by Mr. A to Mr. C
XGBA	German teaching action by Mr. B to Mr. A
XGBC	German teaching action by Mr. B to Mr. C
XFCA	French teaching action by Mr. C to Mr. A
XFCB	French teaching action by Mr. C to Mr. B
QXEAB	Observed value of English teaching action by Mr. A to Mr. B
QXEAC	Observed value of English teaching action by Mr. A to Mr. C
QXGBA	Observed value of German teaching action by Mr. B to Mr. A
QXGBC	Observed value of German teaching action by Mr. B to Mr. C
QXFCA	Observed value of French teaching action by Mr. C to Mr. A
QXFCB	Observed value of French teaching action by Mr. C to Mr. B
SA	Situation of Mr. A
SB	Situation of Mr. B
SC	Situation of Mr. C
AGBA	Need Coefficient of Mr. A for German teaching action by Mr. B
AFCA	Need Coefficient of Mr. A for French teaching action by Mr. C

AEAB	Need Coefficient of Mr. B for English teaching action by Mr. A
AFCB	Need Coefficient of Mr. B for French teaching action by Mr. C
AGBC	Need Coefficient of Mr. C for German teaching action by Mr. B
AEAC	Need Coefficient of Mr. C for English teaching action by Mr. A
MA	Motivation of Mr. A
MB	Motivation of Mr. B
MC	Motivation of Mr. C
KXEAB	Distribution Coefficient of English teaching action to Mr. B
KXEAC	Distribution Coefficient of English teaching action to Mr. C
KXGBA	Distribution Coefficient of German teaching action to Mr. A
KXGBC	Distribution Coefficient of German teaching action to Mr. C
KXFCA	Distribution Coefficient of French teaching action to Mr. A
KXFCB	Distribution Coefficient of French teaching action to Mr. B
BXEAB	Transformation Coefficient of English teaching action to Mr. B
BXEAC	Transformation Coefficient of English teaching action to Mr. C
BXGBA	Transformation Coefficient of German teaching action to Mr. A
BXGBC	Transformation Coefficient of German teaching action to Mr. C
BXFCA	Transformation Coefficient of French teaching action to Mr. A

BXFCB	Transformation Coefficient of French teaching action to Mr. B
DAEAB	Gain of effort for the action XEAB through two way loop
DBEAB	Gain of effort for the action XEAB through three way loop
DAEAC	Gain of effort for the action XEAC through two way loop
DBEAC	Gain of effort for the action XEAC through three way loop
DAGBA	Gain of effort for the action XGBA through two way loop
DBGBA	Gain of effort for the action XGBA through three way loop
DAGBC	Gain of effort for the action XGBC through two way loop
DBGBC	Gain of effort for the action XGBC through three way loop
DAFCA	Gain of effort for the action XFCA through two way loop
DBFCA	Gain of effort for the action XFCA through three way loop
DAFCB	Gain of effort for the action XFCB through two way loop
DBFCB	Gain of effort for the action XFCB through three way loop
DTEAB	Total gain of effort for the action XEAB
DTEAC	Total gain of effort for the action XEAC
DTGBA	Total gain of effort for the action XGBA
DTGBC	Total gain of effort for the action XGBC
DTFCA	Total gain of effort for the action XFCA
DTFCB	Total gain of effort for the action XFCB

SXGBA	German language proficiency of Mr. A
SXFCA	French language proficiency of Mr. A
SXEAB	English language proficiency of Mr. B
SXFCB	French language proficiency of Mr. B
SXGBC	German language proficiency of Mr. C
SXEAC	English language proficiency of Mr. C
LGBA	Decrease of German language proficiency of Mr. A
LFCA	Decrease of French language proficiency of Mr. A
LEAB	Decrease of English language proficiency of Mr. B
LFCB	Decrease of French language proficiency of Mr. B
LGBC	Decrease of German language proficiency of Mr. C
LEAC	Decrease of English language proficiency of Mr. C
AGBAO	Original Need Coefficient for XGBA
AFCAO	Original Need Coefficient for XFCA
AEABO	Original Need Coefficient for XEAB
AFCBO	Original Need Coefficient for XFCB
AGBCO	Original Need Coefficient for XGBC
AEACO	Original Need Coefficient for XEAC
AYGBA	Assumed value of Distribution Coefficient for XGBA
AYFCA	Assumed value of Distribution Coefficient for XFCA
AYEAB	Assumed value of Distribution Coefficient for XEAB
AYFCB	Assumed value of Distribution Coefficient for XFCB
AYGBC	Assumed value of Distribution Coefficient for XGBC
AYEAC	Assumed value of Distribution Coefficient for XEAC
GBAC	Amount of change in situation of Mr. C by the change in motivation of Mr. A produced by originally Mr. B

GCAC	Amount of change in situation of Mr. C by the change in motivation of Mr. A produced by originally Mr. C
GBAB	Amount of change in situation of Mr. B by the change in motivation of Mr. A produced by originally Mr. B
GCAB	Amount of change in situation of Mr. B by the change in motivation of Mr. A produced by originally Mr. C
GABC	Amount of change in situation of Mr. C by the change in motivation of Mr. B produced by originally Mr. A
GCBC	Amount of change in situation of Mr. C by the change in motivation of Mr. B produced by originally Mr. C
GABA	Amount of change in situation of Mr. A by the change in motivation of Mr. B produced by originally Mr. A
GCBA	Amount of change in situation of Mr. A by the change in motivation of Mr. B produced by originally Mr. C
GACB	Amount of change in situation of Mr. B by the change in motivation of Mr. C produced by originally Mr. A
GBCB	Amount of change in situation of Mr. B by the change in motivation of Mr. C produced by originally Mr. B
GACA	Amount of change in situation of Mr. A by the change in motivation of Mr. C produced by originally Mr. A
GBCA	Amount of change in situation of Mr. A by the change in motivation of Mr. C produced by originally Mr. B
UNEAB	Amount of change in situation of Mr. B produced by one unit of effort for XEAB
UNEAC	Amount of change in situation of Mr. C produced by one unit of effort for XEAC
UNGBA	Amount of change in situation of Mr. A produced by one unit of effort for XGBA
UNGBC	Amount of change in situation of Mr. C produced by one unit of effort for XGBC
UNFCA	Amount of change in situation of Mr. A produced by one unit of effort for XFCA
UNFCB	Amount of change in situation of Mr. B produced by one unit of effort for XFCB

XEABO	Original calculated action of XEAB
XEACO	Original calculated action of XEAC
XGBAO	Original calculated action of XGBA
XGBCO	Original calculated action of XGBC
XFCAO	Original calculated action of XFCA
XFCBO	Original calculated action of XFCB
CXAEE	Sum of the total gain of XEAB and XEAC
CXBGG	Sum of the total gain of XGBA and XGBC
CXCFF	Sum of the total gain of XFCA and XFCB
AHGBA	Average delay information 6 times period of KXGBA
BHGBA	Average delay information 2 times period of KXGBA
AHFCA	Average delay information 6 times period of KXFCA
BHFCA	Average delay information 2 times period of KXFCA
AHEAB	Average delay information 6 times period of KXEAB
BHEAB	Average delay information 2 time period of KXEAB
AHFCB	Average delay information 6 times period of KXFCB
BHFCB	Average delay information 2 times period of KXFCB
AHGBC	Average delay information 6 times period of KXGBC
BHGBC	Average delay information 2 times period of KXGBC
AHEAC	Average delay information 6 times period of KXEAC
BHEAC	Average delay information 2 times period of KXEAC
SHGBA	Average change per times period in the value of Distribution Coefficient for action XGBA during 4 unit time
SHFCA	Average change per times period in the value of Distribution Coefficient for action XFCA during 4 unit time

SHEAB Average change per times period in the value of
Distribution Coefficient for action XEAB during 4
unit time

SHFCB Average change per times period in the value of
Distribution Coefficient for action XFCB during 4
unit time

SHGBC Average change per times period in the value of
Distribution Coefficient for action XGBC during 4
unit time

SHEAC Average change per times period in the value of
Distribution Coefficient for action XEAC during 4
unit time

COA Consideration Coefficient for two way loop

COB Consideration Coefficient for three way loop

APPENDIX B COMPUTER PROGRAM

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RUN      TRIAL1
NOTE
NOTE      SIMPLE MODEL EXAMPLE -LEARNING LANGUAGE-
NOTE
NOTE      *****NEED COEFFICIENT *****
NOTE
1L      SXGBA,K=SXGBA,J+(DT)*(XGBA,JK-LGBA,JK)
12R     LGBA,KL=(SXGBA,K)*(0.01)
14A     AGBAO,K=2.0+(SXGBA,K)*(-0.004)
56A     AGBA,K=MAX(0.0,AGBAO,K)
NOTE
1L      SXFCA,K=SXFCA,J+(DT)*(XFCA,JK-LFCA,JK)
12R     LFCA,KL=(SXFCA,K)*(0.01)
14A     AFCAO,K=1.0+(SXFCA,K)*(-0.002)
56A     AFCA,K=MAX(0.0,AFCAO,K)
NOTE
1L      SXEAB,K=SXEAB,J+(DT)*(XEAB,JK-LEAB,JK)
12R     LEAB,KL=(SXEAB,K)*(0.01)
14A     AEABO,K=1.0+(SXEAB,K)*(-0.002)
56A     AEAB,K=MAX(0.0,AEABO,K)
NOTE
1L      SXFCB,K=SXFCB,J+(DT)*(XFCB,JK-LFCB,JK)
12R     LFCB,KL=(SXFCB,K)*(0.01)
14A     AFCBO,K=2.0+(SXFCB,K)*(-0.004)
56A     AFCB,K=MAX(0.0,AFCBO,K)
NOTE
1L      SXGBC,K=SXGBC,J+(DT)*(XGBC,JK-LGBC,JK)
12R     LGBC,KL=(SXGBC,K)*(0.01)
14A     AGBCO,K=1.0+(SXGBC,K)*(-0.002)
56A     AGBC,K=MAX(0.0,AGBCO,K)
NOTE
1L      SXEAC,K=SXEAC,J+(DT)*(XEAC,JK-LEAC,JK)
12R     LEAC,KL=(SXEAC,K)*(0.01)
14A     AEACO,K=2.0+(SXEAC,K)*(-0.004)
56A     AEAC,K=MAX(0.0,AEACO,K)
NOTE
NOTE      *****SITUATION FUNCTION*****
NOTE
NOTE      C
NOTE      DEL=1.0
3L      QXGBA,K=QXGBA,J+(DT)*(1/DEL)*(XGBA,JK-QXGBA,J)
3L      QXFCA,K=QXFCA,J+(DT)*(1/DEL)*(XFCA,JK-QXFCA,J)
3L      QXEAB,K=QXEAB,J+(DT)*(1/DEL)*(XEAB,JK-QXEAB,J)
3L      QXFCB,K=QXFCB,J+(DT)*(1/DEL)*(XFCB,JK-QXFCB,J)
3L      QXGBC,K=QXGBC,J+(DT)*(1/DEL)*(XGBC,JK-QXGBC,J)

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Run A

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3L      QXEAC.K=QXEAC.J+(DT)*(1/DEL)*(XEAC.JK-QXEAC.J)
NOTE
15A     SA.K=(AGBA.K)*(QXGBA.K)+(AFCA.K)*(QXFCA.K)
15A     SB.K=(AEAB.K)*(QXEAB.K)+(AFCB.K)*(QXFCB.K)
15A     SC.K=(AGBC.K)*(QXGBC.K)+(AEAC.K)*(QXEAC.K)
NOTE
NOTE    ***** TRANSFER COEFFICIENT *****
NOTE
C       BXEAB=1.0
C       BXEAC=1.0
NOTE
C       BXGBA=1.0
C       BXGBC=1.0
NOTE
C       BXFCA=1.0
C       BXFCB=1.0
NOTE
NOTE    ***** MOTIVATION FUNCTION *****
NOTE
7A      MA.K=SA.K+1.0
7A      MB.K=SB.K+1.0
7A      MC.K=SC.K+1.0
NOTE
NOTE    ***** DECISION FUNCTION *****
NOTE
48A     AYGBA.K=AGBA.K/(AGBA.K+AFCA.K)
48A     AYFCA.K=AFCA.K/(AGBA.K+AFCA.K)
48A     AYEAB.K=AEAB.K/(AEAB.K+AFCB.K)
48A     AYFCB.K=AFCB.K/(AEAB.K+AFCB.K)
48A     AYGBC.K=AGBC.K/(AGBC.K+AEAC.K)
48A     AYEAC.K=AEAC.K/(AGBC.K+AEAC.K)
NOTE
13A     GBAC.K=(AYGBA.K)*(BXEAC)*(AEAC.K)
13A     GCAC.K=(AYFCA.K)*(BXEAC)*(AEAC.K)
13A     GHAB.K=(AYGBA.K)*(BXEAB)*(AEAB.K)
13A     GCAH.K=(AYFCA.K)*(BXEAB)*(AEAB.K)
NOTE
13A     GABC.K=(AYEAB.K)*(BXGBC)*(AGBC.K)
13A     GCBC.K=(AYFCB.K)*(BXGBC)*(AGBC.K)
13A     GABA.K=(AYEAB.K)*(BXGBA)*(AGBA.K)
13A     GGBA.K=(AYFCB.K)*(BXGBA)*(AGBA.K)
NOTE
13A     GACB.K=(AYEAC.K)*(BXFCB)*(AFCB.K)
13A     GBCB.K=(AYGBC.K)*(BXFCB)*(AFCB.K)
13A     GACA.K=(AYEAC.K)*(BXFCA)*(AFCA.K)
13A     GBCA.K=(AYGBC.K)*(BXFCA)*(AFCA.K)
NOTE
12A     UNEAB.K=(BXEAB)*(AEAB.K)
12A     UNEAC.K=(BXEAC)*(AEAC.K)
12A     UNGBA.K=(BXGBA)*(AGBA.K)
12A     UNGBC.K=(BXGBC)*(AGBC.K)
12A     UNFCA.K=(BXFCA)*(AFCA.K)
12A     UNFCB.K=(BXFCB)*(AFCB.K)
NOTE
NOTE    ***** FOR -4- *****
NOTE
13A     DAEAB.K=(UNEAB.K)*(GAHA.K)*(1.0)

```

Run A (continued)

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13A D1EAB.K=(UNEAB,K)(GABC,K)(GBCA,K)
12A DBEAB.K=(D1EAB,K)(0.8)
NOTE
7A DTEAB.K=DAEAB,K+DBEAB,K
NOTE
13A DAEAC.K=(UNEAC,K)(GACA,K)(1.0)
13A D1EAC.K=(UNEAC,K)(GACB,K)(GCPA,K)
12A DBEAC.K=(D1EAC,K)(0.8)
NOTE
7A DTEAC.K=DAEAC,K+DBEAC,K
NOTE
7A CXAEE.K=DTEAB,K+DTEAC,K
NOTE
20A KXEAB.K=DTEAB,K/CXAEE,K
20A KXEAC.K=DTEAC,K/CXAEE,K
NOTE
13R XEABO.KL=(MA,K)(KXEAB,K)(BXEAB)
54R XEAB.KL=MIN(35.0,XEABO,K)
NOTE
13R XEACO.KL=(MA,K)(KXEAC,K)(BXEAC)
54R XEAC.KL=MIN(35.0,XEACO,K)
NOTE
NOTE ***** FOR -B- *****
NOTE
13A DAGBA.K=(UNGBA,K)(GBAB,K)(1.0)
13A D1GBA.K=(UNGBA,K)(GBAC,K)(GACB,K)
12A DBGBA.K=(D1GBA,K)(0.8)
NOTE
7A DTGBA.K=DAGBA,K+DBGBA,K
NOTE
13A DAGBC.K=(UNGBC,K)(GBCB,K)(1.0)
13A D1GBC.K=(UNGBC,K)(GBCA,K)(GCAB,K)
12A DBGBC.K=(D1GBC,K)(0.8)
NOTE
7A DTGBC.K=DAGBC,K+DBGBC,K
NOTE
7A CXBGG.K=DTGBA,K+DTGBC,K
NOTE
20A KXGBA.K=DTGBA,K/CXBGG,K
20A KXGBC.K=DTGBC,K/CXBGG,K
NOTE
13R XGBAO.KL=(MB,K)(KXGBA,K)(BXGBA)
54R XGBA.KL=MIN(35.0,XGBAO,K)
NOTE
13R XGBCO.KL=(MB,K)(KXGBC,K)(BXGBC)
54R XGBC.KL=MIN(35.0,XGBCO,K)
NOTE
NOTE ***** FOR -C- *****
NOTE
13A DAFCA.K=(UNFCA,K)(GCAC,K)(1.0)
13A D1FCA.K=(UNFCA,K)(GCAB,K)(GAPC,K)
12A DBFCA.K=(D1FCA,K)(0.8)
NOTE
7A DTFCA.K=DAFCA,K+DBFCA,K
NOTE
13A DAFCB.K=(UNFCB,K)(GCBK,K)(1.0)
13A D1FCB.K=(UNFCB,K)(GCPA,K)(GBAC,K)

```

Run A (continued)

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12A DBFCB,K=(D1FCB,K) (0.0)
NOTE
7A DTFCB,K=DAFCB,K+DBFCB,K
NOTE
7A CXCFB,K=DTFCA,K+DTFCB,K
NOTE
20A KXFCA,K=DTFCA,K/CXCFB,K
20A KXFCB,K=DTFCB,K/CXCFB,K
NOTE
13R XFCAO,KL=(MC,K) (KXFCA,K) (BXFCA)
54R XFCA,KL=MIN(35.0,XFCAO,K)
NOTE
13R XFCBO,KL=(MC,K) (KXFCB,K) (BXFCB)
54R XFCB,KL=MIN(35.0,XFCBO,K)
NOTE
NOTE ***** INITIAL CONDITION *****
NOTE
6N QXGB=C7
6N QXFC=C8
6N QXEAB=C9
6N QXFCB=C10
6N QXGBC=C11
6N QXEAC=C12
NOTE
6N SXGBA=C1
6N SXFCA=C2
6N SXEAB=C3
6N SXFCB=C4
6N SXGBC=C5
6N SXEAC=C6
NOTE
C C1=0.0
C C2=0.0
C C3=0.0
C C4=0.0
C C5=0.0
C C6=0.0
NOTE
C C7=0.0
C C8=0.0
C C9=0.0
C C10=0.0
C C11=0.0
C C12=0.0
NOTE
NOTE
PRINT 1)SA,AGBA,AFCA/2)KXEAB,KXEAC,XEAB,XEAC/3)SB,AEAB,AFCB/4)KXGBA,KXGB
X1 C,XGBA,XGBC/5)SC,AGBC,AEAC/6)KXFCA,KXFCB,XFCA,XFCB
NOTE
PLOT XEAB=A,XEAC=B,XGBA=C,XGBC=D,XFCA=E,XFCB=F
NOTE
PLOT SXEAB=A,SXEAC=B,SXGBA=C,SXGBC=D,SXFCA=E,SXFCB=F
NOTE
PLOT KXEAB=A,KXEAC=B,KXGBA=C,KXGBC=D,KXFCA=E,KXFCB=F
NOTE
PLOT SA=A,SB=B,SC=C
NOTE

```

Run A (continued)

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```
SPEC DT=0.1/LENGTH=100/PRTPER=1.0/PLTPER=1.0
NOTE
RUN TRIAL2
C C1=50.0
C C2=50.0
C C3=50.0
C C4=50.0
C C5=50.0
C C6=50.0
NOTE
RUN TRIAL3
C C1=100.0
C C2=50.0
C C3=50.0
C C4=100.0
C C5=50.0
C C6=100.0
NOTE
RUN TRIAL4
C C1=50.0
C C2=100.0
C C3=100.0
C C4=50.0
C C5=100.0
C C6=50.0
NOTE
END
```

Run A (continued)

A.D.C.: O/V (Observed value of Distribution Coefficient)

```

NOTE
NOTE *****DECISION FUNCTION *****
NOTE
3L  AYGPA.K=AYGBA.J+(NT)*(1/DEL1)*(KXGRA.J-AYGPA.J)
3L  AYFCA.K=AYFCA.J+(NT)*(1/DEL1)*(KXFCA.J-AYFCA.J)
3L  AYEAB.K=AYEAB.J+(NT)*(1/DEL1)*(KXEAB.J-AYEAB.J)
3L  AYFCB.K=AYFCB.J+(NT)*(1/DEL1)*(KXFCB.J-AYFCB.J)
3L  AYGHC.K=AYGHC.J+(NT)*(1/DEL1)*(KXGHC.J-AYGHC.J)
3L  AYEAC.K=AYEAC.J+(NT)*(1/DEL1)*(KXEAC.J-AYEAC.J)
NOTE

```

A.D.C.: F/V (Forecasted value of Distribution Coefficient)

```

NOTE
NOTE *****DECISION FUNCTION *****
NOTE
NOTE
C      DEL2=6.0
C      DEL3=2.0
NOTE
3L     AHGBA.K=AHGBA.J+(DT)*(1/DEL2)*(KXGBA.J-AHGBA.J)
3L     RHGBA.K=RHGBA.J+(DT)*(1/DEL3)*(KXGBA.J-RHGBA.J)
7A     SHGBA.K=RHGBA.K-AHGBA.K
14A    AYGBA.K=KXGBA.J+(SHGBA.K)*(0.25)
NOTE
3L     AHFCA.K=AHFCA.J+(DT)*(1/DEL2)*(KXFCA.J-AHFCA.J)
3L     RHFCA.K=RHFCJ.J+(DT)*(1/DEL3)*(KXFCA.J-BHFCA.J)
7A     SHFCA.K=RHFCJ.K-AHFCA.K
14A    AYFCA.K=KXFCA.J+(SHFCA.K)*(0.25)
NOTE
3L     AHEAB.K=AHEAB.J+(DT)*(1/DEL2)*(KXFAB.J-AHEAB.J)
3L     RHEAB.K=AHEAB.J+(DT)*(1/DEL3)*(KXEAB.J-BHEAB.J)
7A     SHEAB.K=RHEAB.K-AHEAB.K
14A    AYEAB.K=KXEAB.J+(SHEAB.K)*(0.25)
NOTE
3L     AHFCB.K=AHFCB.J+(DT)*(1/DEL2)*(KXFCEB.J-AHFCB.J)
3L     RHFCB.K=RHFCB.J+(DT)*(1/DEL3)*(KXFCEB.J-BHFCB.J)
7A     SHFCB.K=RHFCB.K-AHFCB.K
14A    AYFCB.K=KXFCEB.J+(SHFCB.K)*(0.25)
NOTE
3L     AHGBC.K=AHGBC.J+(DT)*(1/DEL2)*(KXGBC.J-AHGBC.J)
3L     RHGBC.K=RHGBC.J+(DT)*(1/DEL3)*(KXGBC.J-BHGBC.J)
7A     SHGBC.K=RHGBC.K-AHGBC.K
14A    AYGBC.K=KXGBC.J+(SHGBC.K)*(0.25)
NOTE
3L     AHEAC.K=AHEAC.J+(DT)*(1/DEL2)*(KXEAC.J-AHEAC.J)
3L     RHEAC.K=RHEAC.J+(DT)*(1/DEL3)*(KXEAC.J-BHEAC.J)
7A     SHEAC.K=RHEAC.K-AHEAC.K
14A    AYEAC.K=KXEAC.J+(SHEAC.K)*(0.25)
NOTE
NOTE

```


N/F: 1,1 (Same Need Coefficient)

```

NOTE
NOTE *****NEED COEFFICIENT *****
NOTE
1L  SXGBA,K=SYGBA,J+(DT)*(XGBA,JK-LGBA,JK)
12R  LGBA,KL=(SXGBA,K)*(0.01)
14A  AGBAO,K=1.0+(SXGBA,K)*(-0.002)
56A  AGBA,K=MAX(0.0,AGBAO,K)
NOTE
1L  SXFCA,K=SXFCA,J+(DT)*(XFCA,JK-LFCA,JK)
12R  LFCA,KL=(SXFCA,K)*(0.01)
14A  AFCAO,K=1.0+(SXFCA,K)*(-0.002)
56A  AFCA,K=MAX(0.0,AFCAO,K)
NOTE
1L  SXEAB,K=SXEAB,J+(DT)*(XEAB,JK-LEAB,JK)
12R  LEAB,KL=(SXEAB,K)*(0.01)
14A  AEABO,K=1.0+(SXEAB,K)*(-0.002)
56A  AEAB,K=MAX(0.0,AEABO,K)
NOTE
1L  SXFCB,K=SXFCB,J+(DT)*(XFCB,JK-LFCB,JK)
12R  LFCB,KL=(SXFCB,K)*(0.01)
14A  AFCHO,K=1.0+(SXFCB,K)*(-0.002)
56A  AFCB,K=MAX(0.0,AFCBO,K)
NOTE
1L  SXGBC,K=SXGBC,J+(DT)*(XGBC,JK-LGBC,JK)
12R  LGBC,KL=(SXGBC,K)*(0.01)
14A  AGBCO,K=1.0+(SXGBC,K)*(-0.002)
56A  AGBC,K=MAX(0.0,AGBCO,K)
NOTE
1L  SXEAC,K=SXEAC,J+(DT)*(XEAC,JK-LEAC,JK)
12R  LEAC,KL=(SXEAC,K)*(0.01)
14A  AEACO,K=1.0+(SXEAC,K)*(-0.002)
56A  AEAC,K=MAX(0.0,AEACO,K)
NOTE

```

P I/C (Different initial value of proficiency)

```
NOTE
C      C1=0.0
C      C2=0.0
C      C3=0.0
C      C4=0.0
C      C5=0.0
C      C6=0.0
NOTE
RUN    TRIAL3
NOTE
C      C1=50.0
C      C2=100.0
C      C3=100.0
C      C4=50.0
C      C5=100.0
C      C6=50.0
NOTE
RUN    TRIAL4
C      C1=100.0
C      C2=50.0
C      C3=50.0
C      C4=100.0
C      C5=50.0
C      C6=100.0
RUN    TRIAL5
C      C1=50.0
C      C2=50.0
C      C3=50.0
C      C4=50.0
C      C5=50.0
C      C6=50.0
NOTE
END
```

Run G (continued)

C/C (Different Consideration Coefficient)

```

NOTE *****FOR -A- *****
NOTE
13A DAEAB.K=(UNEAB.K)(GABA.K)(COA)
13A D1EAB.K=(UNEAB.K)(GABC.K)(GHCA.K)
12A DBEAB.K=(D1EAB.K)(COB)
NOTE
13A DAEAC.K=(UNEAC.K)(GACA.K)(COA)
13A D1EAC.K=(UNEAC.K)(GACB.K)(GCBA.K)
12A DBEAC.K=(D1EAC.K)(COB)
NOTE
NOTE ***** FOR -B- *****
NOTE
13A DABBA.K=(UNGBA.K)(GBAB.K)(COA)
13A D1GBA.K=(UNGBA.K)(GBAC.K)(GACB.K)
12A DBGBA.K=(D1GBA.K)(COB)
NOTE
13A DABBC.K=(UNGBB.K)(GBCB.K)(COA)
13A D1GBB.K=(UNGBB.K)(GBCA.K)(GACB.K)
12A DBGBB.K=(D1GBB.K)(COB)
NOTE
NOTE ***** FOR -C- *****
NOTE
13A DAFCA.K=(UNFCA.K)(GCAC.K)(COA)
13A D1FCA.K=(UNFCA.K)(GCAB.K)(GABC.K)
12A DBFCA.K=(D1FCA.K)(COB)
NOTE
13A DAFCB.K=(UNFCB.K)(GCBF.K)(COA)
13A D1FCB.K=(UNFCB.K)(GCBF.K)(GBAC.K)
12A DBFCB.K=(D1FCB.K)(COB)
NOTE
C COA=1.0
C COB=1.0
NOTE
RUN TRIAL3
C COB=0.8
NOTE
RUN TRIAL4
C COB=0.5
NOTE
RUN TRIAL5
C COB=0.0
NOTE
END

```

M/F; B (Motivation Function Type B)

```

NOTE
NOTE ***** MOTIVATION FUNCTION *****
NOTE
14A MA.K=9.8+(SA.K)(0.23)
14A MB.K=9.8+(SB.K)(0.23)
14A MC.K=9.8+(SC.K)(0.23)
NOTE

```

BIBLIOGRAPHY

1. Allen, R. G. D., Mathematical Economics, Macmillan St. Martin's Press, 1970, page 493.
2. Forrester, Jay W., Industrial Dynamics, The M.I.T. Press, Cambridge, Massachusetts, 1961.
3. Forrester, Jay W., Urban Dynamics, M.I.T. Press, Cambridge, Massachusetts, 1969.
4. Forrester, Jay W., World Dynamics, Wright-Allen Press, Inc., 1971.
5. Hampton, David R., Charles E. Summer and Ross A. Webber, Organizational Behavior and the Practice of Management, Scott, Foresman and Company, 1973, page 155.
6. Henderson, James M. and Richard E. Quandt, Microeconomic Theory A Mathematical Approach, McGraw-Hill Book Company, 1971.
7. Hardar, Josef, Mathematical Theory of Economic Behavior, Addison-Wesley Publishing Company, 1971.
8. Pugh, Alexander L., III, Dynamo User's Manual, 2nd Ed., The M.I.T. Press, Cambridge, Massachusetts, 1973.